The Birth of a Multinational

Innovation and Foreign Acquisitions *

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Abstract

This paper studies a firm's joint decision to innovate and become a multinational, and what it implies for productivity at home. Using panel data of Spanish firms with detailed information on both their innovation and international activities, we show that innovation is a lumpy and disruptive process: it occurs sporadically and is followed by an immediate drop in sales growth. We incorporate this technological feature into a continuous-time stochastic model of the firm, which endogenously decides if and when to expand internationally. Through the lens of this model, we quantify the impact of foreign investment on the innovation decision at the headquarter firm, and the transfer of the headquarter's productivity to the affiliated foreign unit. We find that the option to become a multinational increases the productivity of the domestic headquarter firm by 16%.

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1 Introduction

The 20^{th} century was the century of the modern multinational corporation. The industrial revolution and the decline in transport, information and communication costs allowed them to go from virtual non-existence around 1870 to dominating not just international trade flows but also domestic production in the 21^{st} century: by 2019, U.S. parent companies accounted for 22% of total private sector employment, and 23% of total private sector value added in the United States.[1] Similarly, European parent companies accounted for 21% of employment in the E.U. in 2022. However, in spite of this dramatic growth worldwide, and its well documented impact on destination countries, much less is known about how much it contributed to domestic growth and productivity in the parent firm's home country. This gap is increasingly consequential: as protectionism resurges in the 21st century, and policies seek to restrict firms' foreign expansion and repatriate production, it becomes essential to understand how such measures reshape firms' incentives to innovate and grow at home.

Multinationals (MNCs) are at the top of the cross-sectional distribution of productivities (see, among others, Mayer and Ottaviano, 2007, Bernard, Jensen, Redding, and Schott, 2018, Antras and Yeaple, 2014 and Bloom and Van Rennen, 2010), but these firms were not always multinational, nor did they start with such superior productivity level. We argue that one cannot understand the dynamics of becoming a multinational—and hence the impact of multinational production on innovation—without recognizing that the two decisions, whether to innovate and whether to become a multinational, are interdependent. Moreover, the innovation and productivity upgrades that eventually allow firms to expand abroad begin well before we observe the birth of the multinational, so that focusing on the cross section, or even on the years just around the foreign expansion, understates the productivity effects of the option to expand across borders. This paper studies how the option of becoming a multinational—a firm that serves other markets through owned foreign subsidiaries—affects domestic productivity and the incentives to innovate. This paper studies how the option of becoming a multinational—a firm that serves other markets through owned foreign subsidiaries—affects domestic productivity and the incentives to innovate.

To analyze this joint process of innovation and entry into multinational activities, we proceed

[1] Source: U.S. Bureau of Economic Analysis, available at `https://www.bea.gov/sites/default/files/papers/BEA-WP2022-11.pdf`.

in two steps. First, we empirically depart from cross-sectional analysis and exploit the panel dimension of the Encuesta Sobre Estrategias Empresariales (ESEE) dataset to document stylized facts of innovation and foreign acquisition dynamics of firms over a twenty-year period.[2] Second, we use this information to develop and calibrate a dynamic model of innovation and sales growth. Through the lens of this model, we quantify the productivity gains in the source-country firm (the headquarters) induced by the opening of its first foreign affiliate. As we discuss below, ignoring the forward-looking dynamics we model would provide biased estimates of the effects of multinational expansion on domestic innovation and productivity.

The ESEE dataset is uniquely suited to our purposes in two respects. It records detailed data on firms' innovation activities, so that we directly observe the productivity-enhancing actions taken within the firm—both product and process innovations. Furthermore, its long panel dimension, combined with information on foreign acquisitions collected since 2000, lets us track not only innovation dynamics over 23 years but also the firm's first foreign acquisition—the moment it becomes a Baby Multinational (*Baby MNC*). This allows us to empirically characterize the innovation and growth patterns of the domestic firm before and after investing in a foreign affiliate for the first time.

We model innovation in a continuous-time stochastic environment, following the literature on vintage capital and optimal investment choice in Abel (1983), Bar-Ilan and Blinder (1992) and Parente (1994). In our framework, innovation is a decision to upgrade the capacity of the firm discretely. This modeling choice is supported by the data: in the sample, the average firm innovates in process 33% of the time, and innovates in products 17% of the years we observe it. Innovation is also disruptive in the sense that it precedes a drop in the firm's growth. This empirical characterization, extensively corroborated in the literature (see, for example, Cooper, Haltiwanger, and Power, 1999 and Klenow, 1998, or Atkin, Chaudhry, Chaudry, and Khandelwal, 2016 in an experimental setting), disciplines our theoretical framework and leads us to model innovation as an option.

The domestic firm's problem is to choose when and how much to innovateand, eventually,

[2]We use the ESEE, a dataset of Spanish manufacturing, because it collects information on many firm dimensions simultaneously including innovation, domestic investment, employment, production and exports. It covers around 1,800 firms annually since 1990, and collects foreign acquisition information since 2000. It was also used, for example, in Guadalupe, Kuzmina, and Thomas, 2012.

whether to expand into multinational activities. We restrict the theoretical analysis to horizontal FDI because, in our sample, the vast majority of the multinational's first affiliates are established to sell in the host country. In this respect, our life-cycle description of the firm is most similar to Garetto, Oldenski, and Ramondo (2017), which also models horizontal FDI as an *option*, though in their model productivity follows an exogenous random process.

The defining element of horizontal FDI is that it relocates production across borders: the firm can produce and sell in the newly entered foreign market rather than exporting from the domestic one. This generates the central trade-off in our model: substituting headquarters' (HQ) exports for foreign affiliate sales reduces the HQs incentives to innovate –unless home technology is transferable within the multinational corporation. The overall effect of multinational activity on the incentive to innovate at home thus depends on the relative strength of two forces: the substitutability of production versus the transferability of technology across units within the corporation. Our model allows us to quantify this trade-off. [3]

We find that, overall, multinational expansion raises the profitability of HQ's innovation: the transferability of HQ technology to the foreign affiliate more than offsets the drop in exports, so that FDI enlarges the market size spanned by HQ technology. This transferability of technology within the corporation is consistent with the findings in Bilir and Morales (2016) and is a necessary condition for the positive assortative matching between firm productivity and multinational activities. In our calibrated model, multinational expansion raises HQ productivity by 16%. This productivity growth translates only partially into HQ output growth (7%). This is because HQ exports are 9% lower than absent FDI.

Our dynamic model also generates novel predictions that the richness of our data allow us to confirm empirically: (1) firms that eventually become multinationals innovate more and grow faster than those that remain domestic (generating positive self-selection endogenously); (2) the domestic HQ is more likely to innovate *after* becoming a multinational than firms that remain domestic; and (3) although in the cross section, the home units of a multinational corporation are larger and more productive than domestic firms, for a given firm sales growth at the HQ increases *before* becoming

[3] Whether foreign-affiliate activity substitutes for or complements home-country exports is itself the subject of recent work (Antràs, Fadeev, Fort, and Tintelnot, 2026; and on export-platform FDI specifically, Antràs, Fadeev, Fort, and Tintelnot, 2024); in our setting this substitution is what dampens — but, given technology transferability, does not eliminate — the HQ's incentive to innovate.

a multinational and drops immediately *after*.

These results, which characterize the joint process of innovation and internationalization, highlight the difficulty of identifying the causal effect of multinational activities on productivity empirically. In our dynamic setting, firms expand abroad precisely after a run of favorable productivity shocks, which then, on average, revert to the mean once the firm has entered. As a result, even a researcher who could perfectly control for initial firm characteristics—comparing productivity and sales growth around the time of entry—would observe a productivity drop at the source-country unit and wrongly conclude that acquiring a foreign affiliate lowers headquarters' productivity. Positive self-selection thus biases naive estimates of the effect of multinational activity on headquarters' innovation and growth downward, not upward. Our strategy is to use the structure of the calibrated model, which accounts for these dynamics and the different sources of selection and bias.

Our paper is related to the literature on exporting firms. The spirit and motivation of our analysis are close to Atkenson and Burstein (2008) and Constantini and Melitz (2007), who model the dynamic joint decision of exporting and innovating. Recent evidence also suggests that innovation is endogenous to the decision to participate in export markets (see, for example, Aw, Roberts, and Winston (2007), Verhoogen (2008), Lileeva and Trefler (2010), Bustos (2011), De Loecker (2011)). A crucial difference is our focus on FDI rather than exporting. Although both international activities have obvious parallels, they differ in a fundamental way: exports do not give rise to the trade-off highlighted earlier between the substitutability of production and the transferability of technology across units within the corporation . Therefore, the conclusions with respect to innovation and productivity found in the export literature do not necessarily apply to multinational activity.

In fact, our data also capture exports and we find that the pattern of dynamic selection around the time of entry into exporting is very different from that around becoming a multinational. Spanish firms in the sample start exporting very small amounts (overwhelmingly within the EU), and relative to non-exporters neither their growth path nor their probability of innovating changes around the time of entry. This is incompatible with a sunk cost of entry into exporting, and as a result, exporting cannot be characterized as a real option.

Finally, our results also complement the large literature on the effect of FDI on host market productivity and innovation. Substantial evidence documents what happens when a firm becomes a subsidiary of a multinational and how this affects the productivity, innovation, and growth of destination firms and host countries more generally (e.g. Guadalupe, Kuzmina, and Thomas, 2012; Arnold and Javorcik, 2009; Alfaro-Uren, Manelici, and Vasquez, 2021). However, much less is known about the effect of internationalization on the production units in the source country. From a policy perspective, one could be concerned that the reallocation of production toward foreign affiliates (rather than exporting from headquarters) reduces the size of the source countrys most productive firms. As we show, this partial view ignores the positive effect of multinational expansion on innovation and productivity at headquarters: we find that the net effect of the option to become a multinational in the home country is positive. Our paper is a first step toward assessing the full effect of this option on domestic markets, taking the different relevant dynamics into account.

The rest of the paper proceeds as follows. Section 2 describes the data and novel stylized facts that will discipline the model. In Section 3, we present a dynamic model in which firms can decide whether to innovate and become multinationals. In Section 4, we calibrate the model and present quantitative simulations and counterfactual exercises. Finally, Section 5 concludes.

2 The Data

2.1 Characteristics of Baby Multinationals in the ESEE

The dataset used for our analysis is the Encuesta Sobre Estrategias Empresariales (ESEE), a panel dataset of Spanish manufacturing firms collected by the Fundación SEPI every year since 1990. It is designed to be representative of the population of Spanish manufacturing firms and includes around 1,800 firms per year (aiming to survey all firms with more than 200 employees and a stratified sample of smaller firms) and categorizes firms into 20 industries, based on the two-digit NACE classification. Since 2000 all firms are asked about their investments in foreign affiliates. Our sample covers the 2000-2022 period.[4]

We define a firm as being a multinational as in Markusen (2002), that is, if it reports to hold at least 50% of the capital of a firm in a country other than Spain (in our sample, 94 percent of firms report to have either 0- or 100-percent stakes in the foreign firm, so our results are not sensitive to this definition–see Figure A.1 in the Appendix). [5]

[4] Details on the survey characteristics and data access guidelines can be obtained at `http://www.funep.es/esee/sp/sinfo_que_es.asp`.

[5] The dataset does not record whether the investment was in an existing company or a greenfield investment,

The panel nature of the data is what allows us to analyze *Baby Multinationals* (*Baby MNCs*): firms that report no foreign holdings in year 2000 or in the first year we observe them and eventually acquire a foreign subsidiary. 90 percent of the firms report no foreign holdings in their first year in the sample and of these, 280 of them acquire their first foreign affiliate over the sample period.

We find that Spanish *Baby MNCs* in the ESEE display patterns of internationalization similar to those described in Conconi, Sapir, and Zanardi (2015) and Gumpert, Moxnes, Ramondo, and Tintelnot (2017). First, most of them begin entering markets that are geographically or culturally closer, 50 percent in the EU, 15 percent in other Spanish-speaking countries (Table A.2 in the Appendix). Second, 87 percent acquire a majority stake in the companies they invest in (Figure A.1 in the appendix). Third, once firms become a *Baby MNC* they are less likely to exit that status than those who first begin to export (Figure A.2 in the Appendix). These facts will motivate our modeling choices, but unlike the stylized facts on innovation of *Baby MNCs*, they are not new.

Firms are also asked about the motive for the foreign acquisition. Table 1 shows the (non-mutually exclusive) motives reported. They clearly indicate that the main motive is a horizontal investment, to supply the foreign market either by manufacturing a product similar to the HQ's, or for resale and distribution (note only 5-7% of firms report the residual category, which by elimination would include the vertical motive "supply inputs to the firm").

Table 1: Motive of Investment in Foreign Affiliate

	1st Investment (1)	Mature MNCs (2)
Manufacture similar product as HQ	49%	52%
Resale and distribution	50%	43%
Adapt and assemble HQ's product	17%	22%
Other (Supply inputs to HQ)	5%	7%

Note: The distribution of motives for the first investment corresponds to the FDI motive reported by *Baby MNCs* (firms that do not have foreign affiliates when we first observe them) in the first year they report having a foreign affiliate. The distribution of motives for Mature MNCs (firms that have foreign affiliates when we firm observe them) is calculated across all observations for which the firm reports a stake in a foreign affiliate of at least 50%. Firms can report more than one motive for the foreign affiliate.

Table 2 presents summary statistics for the full sample (columns 1 to 3), and also according

although we know from other sources that over 90 percent of investments are acquisitions of existing companies (Barba Navaretti and Venables, 2004).

to the multinational status of the firms in: *Baby MNCs* (columns 4 to 6), *Always Domestic* firms (firms that remain domestic during the whole sample period, columns 7 to 9), and *Mature MNCs* (firms that are MNCs from the first time we observe them, columns 10 to 12). In all cases, the figures correspond to the production units located in Spain, the headquarters, and not to the entire multinational corporation.

In addition to recording information on investments abroad, what is unique about our dataset is that it also reports a large number of variables that reflect firms' productivity-enhancing innovation activity. The data include variables that indicate whether the firm conducted process innovation (investment in new machines and/or new forms of organizing) and/or product innovation (products with new materials or new functions, which could involve upgrading the quality of existing products or developing new products). These are the same innovation variables used by Guadalupe, Kuzmina, and Thomas (2012) or Cassiman and Vanormelingen (2013). Note that all innovations captured in the data relate to those that take place in the domestic unit (i.e., the Spanish headquarter). Table 2 describes the fraction of years in which each group of firms report performing product and/or process innovation. *Baby MNCs* innovate more often than firms that stay domestic throughout the sample, and slightly less often than mature MNCs.

We define two different firm performance variables. The first is the natural log of the firm's real sales (similar to Verhoogen, 2008). The second is labor productivity, defined as the natural logarithm of real value added per worker (similar to Lileeva and Trefler, 2010). Columns 5, 8, and 11 of Table 2 show that the median firm that will become a multinational during our sample period is substantially larger than the median firm that remains domestic (€43m vs. €4m), and somewhat smaller than the domestic units of Mature MNCs (€43m vs. €54m). On other characteristics such as employment, capital, average wage, exporting activity and labor productivity Baby MNCs also rank above always domestic firms and are comparable – though ranking generally below – Mature MNCs.

Table 2: Descriptive Statistics

	Total Sample N firms = 6,068			Baby MNC N firms = 280			Always Domestic N firms = 5,242			Mature MNC N firms = 546		
	Mean (1)	Median (2)	SD (3)	Mean (4)	Median (5)	SD (6)	Mean (7)	Median (8)	SD (9)	Mean (10)	Median (11)	SD (12)
FDI participation	9.63	0.00	28.20	35.65	0.00	44.95	0.00	0.00	0.00	63.69	95.00	42.57
Any innovation	0.39	0.00	0.49	0.55	1.00	0.50	0.34	0.00	0.47	0.61	1.00	0.49
Process innovation	0.33	0.00	0.47	0.47	0.00	0.50	0.29	0.00	0.46	0.51	1.00	0.50
Product innovation	0.17	0.00	0.38	0.32	0.00	0.47	0.13	0.00	0.34	0.37	0.00	0.48
Sales (€Mill)	61.11	6.82	301.48	193.59	42.86	659.17	32.13	4.10	170.57	177.31	54.16	501.12
Average wage (€Th)	35.25	32.80	21.55	42.71	40.85	14.43	33.19	30.51	22.47	45.21	44.18	13.55
Exporter dummy	0.67	1.00	0.47	0.93	1.00	0.26	0.60	1.00	0.49	0.96	1.00	0.20
Export>$0/Sales	0.34	0.24	1.46	0.39	0.36	0.29	0.31	0.20	1.70	0.42	0.42	0.29
$\ln(Capital)$	15.03	14.99	2.12	16.70	16.80	1.64	14.59	14.50	1.99	17.07	16.98	1.45
$\ln(Labor)$	4.10	3.87	1.40	5.34	5.43	1.29	3.78	3.56	1.24	5.57	5.50	1.09
$\ln(Labor Productivity)$	11.86	11.82	0.84	12.32	12.26	0.72	11.74	11.69	0.83	12.37	12.33	0.62

Note: Statistics pooled over all years (2000 to 2022). *Baby MNCs* are firms that do not report any foreign investments at the time they enter the sample but will make one in the future. *Always Domestic* corresponds to firms that do not report any foreign investments at the time they enter the sample and do not change their multinational status (they remain domestic firms). *Mature MNCs* are firms that are multinational since we first observe them in the sample. (%) of Product and Process Innovations respectively indicate the proportion of years in the sample that the firms reported doing the corresponding innovation. Product innovation can be changing the function or design of the product, or the components or materials of the product. Process innovation can be changing the form of organizing the firm or improve the machinery.

2.2 Stylized Facts on the Dynamics of FDI Decisions and Innovation

We present now some novel stylized facts on the innovation dynamics of firms that will eventually become *Baby MNCs* (i.e. firms that we observe in the data going from zero to a positive investment position abroad), in the years leading up to their first FDI. Our aim is to trace how these firms diverge from otherwise-similar firms that remain domestic. The model in Section 3 reproduces all of these facts.

A firm's size, productivity, and multinational status are jointly determined by its innovation and entry decisions and evolve together over time. Controlling for these variables as they unfold would absorb precisely the dynamics we want to document. We therefore hold fixed only the firm's *initial* conditions: we compare future *Baby MNCs* with domestic firms that looked identical when we first observed them, and then let their subsequent trajectories differ. The growth path leading up to entry, and the timing of entry itself, are treated as endogenous outcomes rather than as factors to be controlled away.

Concretely, we match each future *Baby MNC* to always-domestic firms in the first year we observe it, using the propensity-score procedure described below. Because treated and control firms are then observationally identical at baseline, any later differences in innovation and growth reflect the divergent paths of firms that started out alike—one of which went on to expand abroad.

To implement this, we follow the propensity-score reweighting procedure of Guadalupe, Kuzmina, and Thomas (2012) (building on Lechner, 1999; Dehejia and Wahba, 1999; and Busso, DiNardo, and McCrary, 2014). We pool future *Baby MNCs* and always-domestic firms in the first year each enters the sample, and estimate by logit the probability that a firm later becomes a *Baby MNC* as a function of its initial characteristics—labor productivity, exporter status, average wage, capital, and labor. This estimated probability is the propensity score $\hat{p}$ which we use to reweight firms in equation (1) so that treated and control groups are balanced on their baseline characteristics.

Table 3 reports the logit estimates underlying the propensity score. Firms that are initially domestic but later become multinationals start with higher productivity, are more likely to export, and have higher employment for a similar capital stock—that is, they are larger and more labor-intensive—than firms that remain domestic; conditional on these characteristics, they do not pay higher wages. These baseline differences reflect the positive selection into multinational activity familiar from heterogeneous-firm models: the firms that expand abroad are drawn from the top of the productivity distribution. This is precisely the selection in initial conditions our matching procedure addresses, so that the dynamics we document below reflect the process of becoming a multinational rather than baseline differences across firms.

Table 3: Propensity Score Estimation

	Future Baby MNC$_i$ (1)
Initial Productivity$_i$	0.614***
	(0.154)
Initial Average Wage$_i$	-0.013*
	(0.007)
Initial Exporter Status$_i$	0.857***
	(0.164)
Initial ln(K_i)	0.156
	(0.096)
Initial ln(L_i)	0.473***
	(0.156)
Year FE	Yes
Industry FE	Yes
Obs.	5297

Note: Table presents the results of logit regressions estimating the probability that a firm is a future *Baby MNC* based on observable characteristics in the year firms first enter the sample. The sample pools observations from future *Baby MNC* with always domestic firms, in the first year firms are in the sample. Standard errors in parentheses are clustered at the industry level. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Fact 1 *Controlling for initial firm's characteristics, firms that eventually become multinationals innovate more often than those that remain domestic.*

This fact is shown in Table 4 which reports marginal effects from logit regressions of the probability that a firm innovates on whether it will become a *Baby MNC*. Future *Baby MNCs* and always-domestic firms are matched in their first year in the sample through the propensity-score procedure above, and for future *Baby MNCs* we use only the years before entry, while they are still domestic. Column 1 considers any innovation (product or process); column 2, product innovation; and column 3, process innovation. Across all three, future *Baby MNCs* are significantly more likely to innovate in a given year than comparable always-domestic firms. For any innovation, the probability is 0.124 higher for future *Baby MNCs* than for the always-domestic group, whose mean frequency is 0.34 (column 1). The gaps are 0.085 for product innovation (against a mean of 0.13)

Table 4: Frequency of Innovation

	Any Innovation (1)	Product Innovation (2)	Process Innovation (3)
Baby MNC	0.124***	0.085***	0.107***
	(0.026)	(0.027)	(0.028)
Year FE	Yes	Yes	Yes
Obs.	31869	31869	31869

Note: Estimates from logit regressions (marginal effects) comparing the probability of innovation of firms that will become MNC (*Baby MNCs*) with firms that will never become MNC (always domestic firms). Observations related to Future *Baby MNCs* are restricted to the years up to their entry into multinational activities. *Baby MNCs* are matched to always domestic firms in the first year firms are in the sample through the propensity score procedure described in section 2.2. Standard errors in parentheses are clustered at the industry level. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

and 0.107 for process innovation (against a mean of 0.29). In short, future *Baby MNCs* innovate more frequently than always-domestic firms well before they enter FDI.

Fact 2 *Innovation at the headquarters is more likely immediately after the first foreign acquisition.*

We document this in Table 5 where we study the dynamic relationship between innovation and FDI using the following specification:

$$\Delta NInnovation_{it} = \sum_{\tau=t-2}^{t+1} \beta_\tau \, \Delta FDI_{i\tau} + \delta_t + \Delta\epsilon_{it} \tag{1}$$

where $NInnovation_{it}$ is a stock (the cumulative number of innovation episodes up to year t) so that its first difference $\Delta NInnovation_{it}$ is a dummy equal to 1 if firm i innovates in year t. $FDI_{it} \in \{0, 1\}$ equals 1 if the firm holds a majority stake in a foreign affiliate in year t ; therefore, ΔFDI_{it} signals a change in the multinational status. Differencing removes the firm fixed effect of the level equation and the leads and lags of ΔFDI_{it} trace innovation from the year before entry (β_{t+1}) to two years after (β_{t-2}). As above, *Baby MNCs* and always domestic firms are matched on initial characteristics. We estimate equation (1) for any innovation with a standard probit model. For process and product innovations we estimate the equations as a system of two equations using a bivariate probit model, that lets the two firm-level innovation decisions to be correlated (i.e., the standard errors can be correlated across the system of equations).[6]

[6]For robustness, we also estimate linear probability models, with very similar results (see Table A.1 in the ap-

Table 5: Innovation around the Time of Entry into MNC

	Any Innovation$_{it}$ (1)	Product Innovation$_{it}$ (2)	Process Innovation$_{it}$ (3)
ΔFDI_{it+1}	0.051	0.066*	0.016
	(0.065)	(0.034)	(0.066)
ΔFDI_{it}	0.157***	0.127***	0.136***
	(0.046)	(0.041)	(0.047)
ΔFDI_{it-1}	0.237***	0.205***	0.182***
	(0.084)	(0.060)	(0.069)
ΔFDI_{it-2}	0.179**	0.129***	0.099
	(0.071)	(0.044)	(0.069)
Year FE	Yes	Yes	Yes
Obs.	17991	17991	17991

Note: Estimates from probit regressions, estimating the probability of doing any innovation (product or process, column 1), product innovation (column 2), and process innovation (column 3). Columns 2 and 3 are estimated as a system of biprobit regressions. Results correspond to the unconditional marginal effects on probability of innovating. *Baby MNCs* are matched to always domestic firms in the first year firms are in the sample through the propensity score procedure described in section 2.2. Standard errors in parentheses are clustered at the industry level. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 5 shows that, relative to comparable always-domestic firms, *Baby MNCs* are more likely to innovate at home in the year they first invest abroad and in the two years that follow. For any innovation in column 1, the marginal probability is 0.157 larger for *Baby MNCs* relative to always domestic firms in the year of entry, rising to 0.237 and 0.179 in the next two years. Both product (column 2) and process (column 3) innovation follow similar profiles. Innovation thus rises sharply around entry, concentrated in the year of and immediately after the first foreign acquisition.

Fact 3 *Sales growth at the headquarters rises in the run-up to the first foreign acquisition and falls immediately after, relative to firms that remain domestic*

This is shown in Table 6 where we estimate the analog of equation (1) with sales growth as the outcome.

$$\Delta \ln(Sales_{it}) = \sum_{\tau=t-2}^{t+1} \beta_\tau \Delta FDI_{i\tau} + \Delta\epsilon_{it} \tag{2}$$

over four years around the first foreign acquisition. As before, the specification is in first differences and *Baby MNCs* are matched to always-domestic firms in their first year in the sample.

pendix).

Column 1 reports total sales growth. Relative to always-domestic firms, the sales of future *Baby MNCs* grow faster in the run-up to entry (β_{t+1} and β_t positive) and then grow about 7% and 5% more slowly in the two years after entry ($\beta_{t-1} = -0.070$, $\beta_{t-2} = -0.052$). This rise-then-fall profile is the sales-growth counterpart of the dynamic selection we formalize later: entry follows favorable shocks that subsequently revert to the mean.

Columns 2 and 3 decompose total sales into domestic sales ($\ln DomSales_{it}$) and exports ($\ln X_{it}$). Both decline after entry, but the fall is sharper for exports—as expected if foreign-affiliate sales partly substitute for HQ exports. Foreign affiliates do not appear to substitute for imports: column 4 shows that imports, if anything, rise in the year of entry (0.322). Column 5 shows that labor-productivity growth, like sales growth, declines in the two years after entry (-0.085 and -0.038).

Sales growth around entry into export markets follows a different pattern (Appendix Table A.4)). Relative to firms that never export, growth of domestic sales is unchanged at the time of entry (column 2) while total sales rise (column 1), and two years on both are significantly higher than for never-exporters. Exporting is thus a continuous process and does not fit the real-option characterization we develop in the next section.

Table 6: Sales around the Time of Entry into MNC

Dep Var	$\Delta\ln(Sales_{it})$ (1)	$\Delta\ln(DomSales_{it})$ (2)	$\Delta\ln(X_{it})$ (3)	$\Delta\ln(IM_{it})$ (4)	$\Delta\ln(LaborProd.)$ (5)
ΔFDI_{it+1}	0.069***	0.025	0.117	-0.123	0.014
	(0.015)	(0.028)	(0.092)	(0.175)	(0.031)
ΔFDI_{it}	0.256*	0.185	0.247	0.322**	0.063
	(0.128)	(0.153)	(0.179)	(0.120)	(0.043)
ΔFDI_{it-1}	-0.070*	-0.003	-0.219***	-0.144	-0.085**
	(0.038)	(0.026)	(0.073)	(0.094)	(0.038)
ΔFDI_{it-2}	-0.052**	-0.069**	-0.073	-0.061	-0.038**
	(0.022)	(0.031)	(0.066)	(0.137)	(0.017)
Obs.	17991	17891	11235	10605	17991
Year FE	Yes	Yes	Yes	Yes	Yes

Note: Estimates from OLS regressions. *Baby MNCs* are matched to always domestic firms in the first year firms are in the sample through the propensity score procedure described in section 2.2. $DomSales_{it}$ X_{it} and IM_{it} denote domestic sales (i.e., sales - exports), exports, and imports, respectively. Standard errors in parentheses are clustered at the industry-level. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

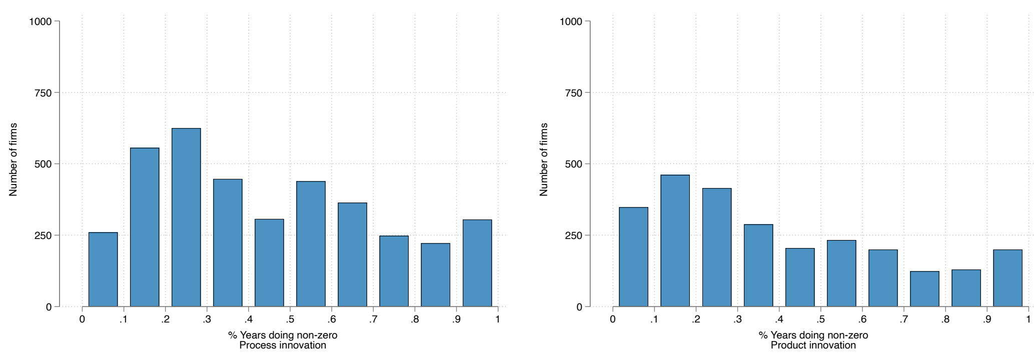
2.3 Characterizing Innovation

Finally, in this section, we use the full sample to document two key properties of innovation in general (not restricted to *Baby MNCs*) that will guide and discipline our model in the next section. These empirical regularities are similar to the findings in Cooper, Haltiwanger, and Power (1999) and Klenow (1998), and we confirm that they are also present in our data.

Fact 4 *Innovation, in either product or process, is lumpy*

Figure 1 illustrates the frequency at which firms do product and process innovation in the sample. Few firms innovate every year. Most do so 20 to 30 percent of the time. The variation in innovation for a firm occurs in the extensive margin (that is, *yes/no*).[7] These innovations expand the technology or product frontier of the firm but not necessarily that of the market.

Figure 1: Firm's Frequency of Innovation



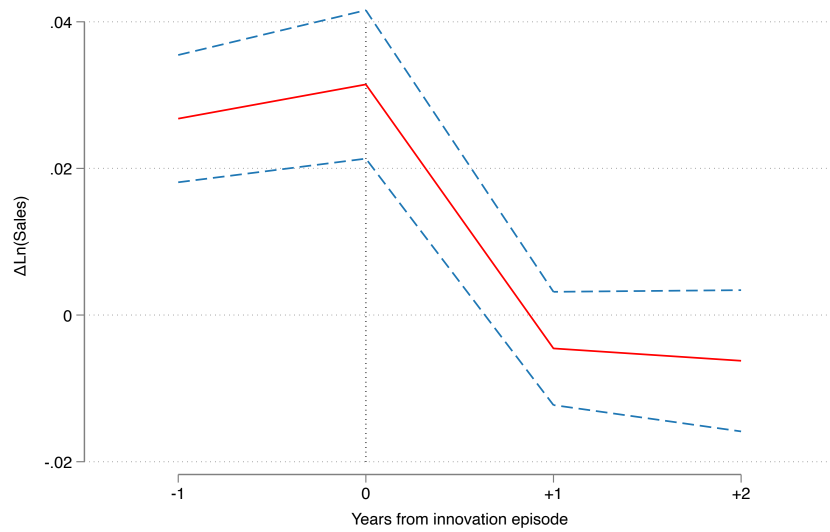
Note: Figures based on the entire ESEE database from 1990 to 2010. Figures display the number of firms that report innovating in process or product in each fraction of years during which they are in the sample as indicated on the X-axis.

Fact 5 *Innovation is disruptive*

Innovation is a disruptive episode in the sense that production growth is higher than average *prior* to the innovation episode, and drops immediately *afterward.* This fact is represented in Figure 2, which plots deviations of *sales growth* relative to its trend (normalized at zero), around the time of innovation, controlling for market-wide conditions (i.e., year fixed effects).

[7]For contrast, Appendix Figure A.3 shows – in a corresponding figure – the fraction of years in which firms report non-zero R&D expenditure. Most firms report doing R&D every year. The variation in R&D occurs at the intensive margin.

Figure 2: Sales Growth and Innovation



Note: Figure based on the entire ESEE sample from 1990 to 2010 (25,072 observations). The figure displays the coefficients on Inn (Any Innovation) from the OLS regression $\Delta \ln(Sales_{it}) = \sum_{\tau=t-2}^{t+1} Inn_{i\tau} + \epsilon_{it}$, with year fixed effects and errors clustered at the industry level. Inn_{it} is a dummy variable equal to 1 if the firm i reports innovation in product or process at time t.

Cooper, Haltiwanger, and Power (1999) and Klenow (1998), among others, also find that investments in capital projects are sporadic and cause disruption. These regularities motivate the modeling of innovation in our theoretical framework.

3 The Model

In this section, we model the life cycle of the firm with a dynamic continuous-time framework that simultaneously accounts for innovation, firm growth, and entry into multinational activities. The goal of a firm is to choose when and by how much to innovate and whether to become a multinational. We will then estimate the relationship between multinational expansion and domestic innovation through the lens of this model to obtain the effect of the option to become a multinational on domestic productivity. Note that we take input prices, the aggregate price index, aggregate expenditure, and the discount rate as exogenous, so the model is in partial equilibrium.

To reflect the fact that innovation is a disruptive lumpy process (see stylized facts in subsection 2.3), we closely follow Pavlova, 2002 and Parente, 1994 and model an innovation episode as the replacement of an old technology vintage with a new one. Under this characterization, innovation becomes an option. The decision to exercise this option takes into account how innovation affects the firm's productivity path and, potentially, the likelihood of a multinational expansion. Then, we allow the domestic firm to choose between two possible states, given by where it locates production: it can stay domestic or become multinational (which is an absorbing state). In both cases, the firm can sell domestically and through exports.

3.1 The Firm

In each instant t, the profits of the domestic firm are determined by a time-varying factor $z_t = z(a_t, h_t)$, which we call *revenue productivity*, and the size of the market, which may include export destinations $j = 1, ..., J$:

$$\pi_d(a_t, h_t, M_d) = z_t \cdot \left(\sum_j^J \tau_j M_j \right) - f \cdot a_t \tag{3}$$

M_j is the size of the market j, which is assumed to be time-invariant, and $\tau_j \leq 1$ is the iceberg cost of export (one for domestic sales $j = 1$). $z(a, h)$ captures both the firm's technological capacity (a_t) and its specific sales shifter (h_t). The technological capacity refers to the productivity of the installed machinery and the sales shifter, which follows an exogenous random process, refers, for example, to the perceived quality of the product or to the firm-specific market penetration.[8] Both components positively affect company profits with decreasing returns. We assume the following functional form:[9]

$$z_t = a_t^{\frac{1}{2}} h_t^{\frac{1}{2}} \tag{4}$$

Higher technological capacity is sustained with a fixed (linear) cost $f \cdot a_t$. This instantaneous fixed cost of production is not needed for any of the qualitative conclusions of the model but helps us to match quantitatively the empirical patterns in the data.

Innovation

Technological capacity cannot be freely accumulated or sold. Instead, the firm upgrades its technology by replacing the old vintage with a new capacity level, a', at a cost proportional to the new

[8]This interpretation is not crucial for the model, and the same logic applies if h has a demand interpretation or is an exogenous productivity component.

[9]These characteristics of the problem are general to any function of the form $z_t = a_t^{\alpha} h_t^{1-\alpha}$, with $\alpha \in (0, 1)$

capacity:

$$F(a') = pa', \tag{5}$$

with $p > 0$. Innovation is therefore lumpy: if the firm innovates at t_n and t_{n+1}, then $a_t = a_n$ for $t \in [t_n, t_{n+1})$.

The sales shifter h_t follows a geometric Brownian motion:

$$\frac{dh_t}{h_t} = \mu\, dt + \sigma\, dW_t, \tag{6}$$

where W_t is a standard Brownian motion, μ is the drift, and $\sigma > 0$ is the volatility. Under this specification, $\ln h_t$ evolves as a random walk. The expected discounted value of profits is finite for $r > (\mu - 1/2\sigma^2)$.

Opening a Foreign Affiliate

The foreign affiliate's productivity combines the HQ's productivity, z_t, with an affiliate-specific component, $z_{f,t}$:

$$z^m_{f,t} = \alpha z_t + z_{f,t}, \tag{7}$$

with $\alpha > 0$. The parameter α measures the transmission of HQ productivity to the affiliate.

Because the affiliate-specific component $z_{f,t}$ does not affect a_t or h_t, the consolidated flow profit after FDI can be written as

$$\Pi_m(a_t, h_t, z_{f,t}) = \pi_m(a_t, h_t) + \gamma(z_{f,t}), \tag{8}$$

with

$$\pi_m(a_t, h_t) \equiv (a_t h_t)^{1/2} M_m - f a_t \qquad \text{and} \qquad M_m \equiv M^m_d + \alpha M^m_f. \tag{9}$$

M_m corresponds to the effective market size served by the HQ's productivity, either directly through HQ sales (M^m_d) or indirectly throught the productivity of the foreign affiliate (αM^m_f). The term $\gamma(z_{f,t})$ collects affiliate-specific profits that are independent of the headquarters' innovation decision.

We assume that multinational expansion increases the HQ-productivity's effective market size:

$$M_m > M_d. \tag{10}$$

This assumption ensures that the gain from becoming a multinational is increasing in HQ productivity and supports the selection of more productive and larger firms into FDI, which is a strong feature of the data.

Finally, beoming a multinational requires a sunk cost. For simplicity, in what follows we refer to F_M as the sunk cost, net of the independent value of the foreign firm.[10]

3.2 The dynamic problem of the firm

The objective of the domestic firm is to choose an innovation plan and a foreign expansion plan. The innovation plan consists of two sets of infinite parameters: the path of technological capacity and the timing of innovation ($\mathbf{a} = \{a_1, a_2, ..., a_n, ..., a_\infty\}$ and $\mathbf{t} = \{t_1, t_2, ..., t_n, ...t_\infty\}$, respectively). The foreign expansion plan is whether and when to acquire a foreign affiliate, T, to maximize the value of the firm, which is given by the expected discounted flow of profits, subject to equations (3)-(6):

$$V_d(a_0, h_0) = \max_{\{\mathbf{t},\mathbf{a},\mathbf{T}\}} E_0 \sum_{n=0}^{\infty} \left[\int_{t_n}^{t_{n+1}} \left[\pi_d(a_n, h_t) + \Delta_\pi(a_n, h_t)\mathbf{1_{t>T}}\right] e^{-rt} dt - a_n p \; e^{-rt_n} \right] - e^{-rT} F_M \quad (11)$$

The entry cost, F_M, is paid at the time of the multinational expansion T. Then, for all $t > T$ ($\mathbf{1_{t>T}}$ is an indicator function equal to one for $t > T$), profits increase in $\Delta_\pi(a_n, h_t)$, defined as the difference between multinational and domestic profits: $\Delta_\pi(a_n, h_t) \equiv (M_m - M_d)(a_n h_t)^{1/2}$.

We do not model the endogenous exit decision from multinational activities nor the acquisition of subsequent foreign affiliates: the multinational option can only be exercised once, so that it is an absorbing state.[11] Although this simplification is not crucial for the conclusions, it makes the model more tractable.

Since being a multinational is an absorbing state, the value of the MNC is simply the expected

---

[10] $F_M \equiv \widetilde{F}_M - \Gamma_f > 0$, where $\widetilde{F}_M$ is the entry cost and Γ_f is the value of the expected flow of profits $\gamma(z_{f,t})$, which is independent of the innovation decisions at HQ.

[11] In our data we would be unable to empirically estimate exit conditions given the low exit rate (see extremely low exit rate in the appendix - Figure A.3), or the impact of the possible acquisition of subsequent foreign affiliates since we cannot cleanly identify further international acquisitions that take place after the first one.

discounted profits when following the optimal innovation policy:[12]

$$V_m(a_0, h_0) = \max_{\{\mathbf{t},\mathbf{a}\}} E_0 \sum_{n=0}^{\infty} \left[\int_{t_n}^{t_{n+1}} \pi_m(a_n, h_t) e^{-rt} dt - a_n p \ e^{-rt_n} \right] \tag{12}$$

3.2.1 MNCs' Optimal Innovation Policy

We begin by solving recursively the optimal innovation policy of the multinational firm described in equation (12). The solution to this problem is similar to that in Parente (1994) and Pavlova (2002). During the region of inaction, when the firm decides not to innovate, the sales of the firm evolve exogenously according to the realization of h, which follows the random process described in equation (6). In the period of inaction, during which the level of technology a is constant, the value of the firm evolves according to the following arbitrage condition:

$$rV_m = \pi_m(a, h) + \frac{1}{dt} E(dV_m) \tag{13}$$

Applying Ito's Lemma:

$$rV_m = \pi_m(a, h) + \mu \ h \ \frac{dV_m}{dh} + \frac{1}{2}\sigma^2 \ h^2 \ \frac{d^2V_m}{dh^2} \tag{14}$$

At the time of innovation, and given the existing technological capacity of the firm a, the level of sales that trigger innovation, given by $h^*(a)$, and the optimal new technological capacity $a^*(a)$ satisfy the following optimality conditions:

1. Value matching. At the time of innovation, given the level of sales h^*, the value of the firm operating under the old technology, a, coincides with the value after adopting the new one, a^*.

$$V_m(h^*, a) = V_m(h^*, a^*) - pa^* \tag{15}$$

2. Smooth pasting condition on level of innovation. Provided that the firm decides to innovate,

[12]The model can be easily expanded to incorporate the option of sequential entry into additional foreign markets. See Gumpert et al. (2017) for a dynamic model along these lines.

its new technological capacity, a^*, is optimal.

$$\frac{dV_m(h^*, a)}{da}\Big|_{a=a^*} - p = 0 \tag{16}$$

3. Smooth pasting condition on timing of innovation. The level of sales that defines the time to abandon the old technology is optimal.

$$\frac{dV_m(h, a)}{dh}\Big|_{h=h^*} - \frac{dV_m(h, a^*)}{dh}\Big|_{h=h^*} = 0 \tag{17}$$

The first condition simply states that there are no *jumps* in the value function at the time of the technology upgrade. The smooth pasting conditions are optimality conditions, i.e., the first-order conditions in the maximization problem.

Profits and the cost of innovation are homogeneous on a. We therefore use the transformation $x \equiv M_m^2 \frac{h}{a}$. The ratio x measures the firm's sales (i.e. $(ah)^{1/2} M_m$) relative to its capacity (i.e. a). We therefore refer to x as *capacity utilization*. The instantaneous flow of profits can be expressed in terms of x, $\pi_m(a, h) = a(x^{1/2} - f)$.

We guess that the value function is $V_m(a, h) = aJ_m(x)$, with the following functional form:

$$J_m(x) = \left[A + Bx^{1/2} + Cx^{\psi}\right] \tag{18}$$

Substituting into (14), we get $A = -f/r$, which corresponds to the discounted fixed cost of production, and $B = 1/\widetilde{r}$, where $\widetilde{r}$ corresponds to the discount factor, adjusted by the random process governing h:

$$\widetilde{r} = r - 1/2(\mu - 1/4\sigma^2) \tag{19}$$

Finally ψ is the positive root of the following fundamental equation, which is larger than 1 for $r > \mu$, which we assume:[13]

$$0 = \psi^2 + \frac{2\mu - \sigma^2}{\sigma^2}\psi - \frac{2r}{\sigma^2} \tag{20}$$

We can rewrite conditions (15), (16), and (17) as functions of capacity utilization $x = M_m^2 h/a$.

[13]The complete mathematical solution is $C_1(a)h^{\psi_1} + C_2(a)h^{\psi_2}$, where ψ_1 and ψ_2 correspond to the positive and negative roots of the fundamental equations. Since the value of the innovation option is null for $h \to 0$, it has to be that $C_2(a) = 0$ and only the positive root enters the solution.

Let x_- and x_+ correspond to capacity utilization just before and after technology upgrade. The optimality conditions above can be rewritten as:

$$J(x_-) = \frac{x_-}{x_+}\left[J(x_+) - p\right] \tag{21}$$

$$x_+ J'(x_+) = J(x_+) - p \tag{22}$$

$$J'(x_-) = J'(x_+) \tag{23}$$

The solution of the problem therefore consists of three unknowns: 1) the capacity utilization that triggers innovation, x_-; 2) the capacity utilization x_+ immediately after the firm optimally expanded the technological capacity; and 3) the constant C in the value function yet to be determined.

Notice that the system of equations is invariant to the state variables. That is, x_-, x_+, and C are independent of the firm's technology or sales (a, h or M_m); they are functions of parameters only. Then, the level of critical capacity utilization that triggers innovation and the improvement in technology when innovation occurs are constant. The following proposition (derived in the appendix) summarizes the optimal innovation policy of the firm

Proposition 1 (Optimal Innovation by MNCs) *Given a market size M_m and technological capacity a, the optimal innovation policy at the HQ is determined by sales, given by $h^*(a)$, such that if $h < h^*(a)$ the firm continues to operate with its existing technology and if h reaches $h^*(a)$, the firm adopts a new technology $a^*(a)$. h^* and a^* are given by:*

$$h^*(a) = \frac{a}{M_m^2}\, x_- \qquad a^*(a) = g\ a$$

where $g = \frac{x_-}{x_+}$ and $x_+ < x_-$ are constants that depend only on parameters r, μ, σ, p and f.

The value function of the multinational firm is given by:

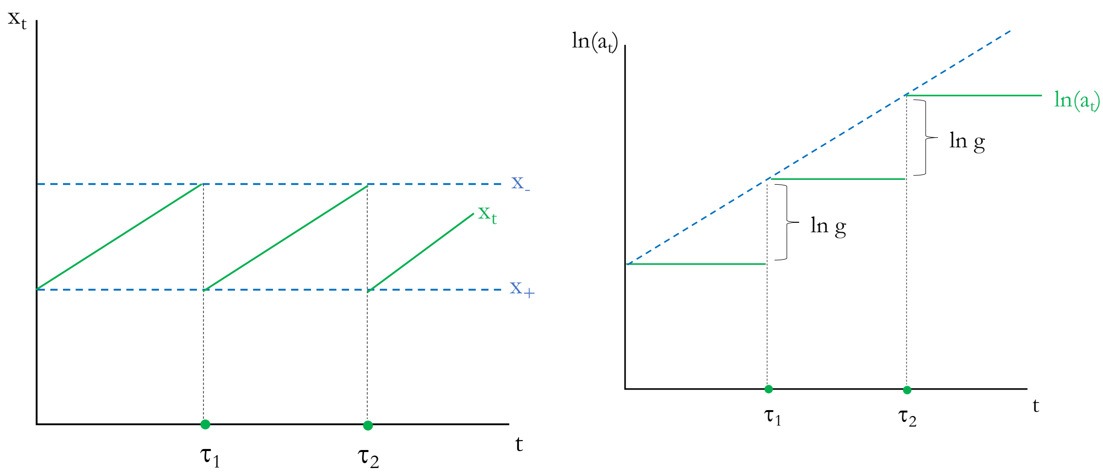
$$V_m(h,a) = \frac{(ha)^{\frac{1}{2}} M_m}{\widetilde{r}} - \frac{fa}{r} + C\ a\ \left(\frac{M_m^2 h}{a}\right)^{\psi} \tag{24}$$

where C is a positive time-invariant constant.

The first two terms in equation (24) correspond to infinitely discounted expected profits. The last term corresponds to the option to innovate. This term is higher if the current technological capacity, a, is low relative to the rest of the variables that define total sales (h and M). At the time of innovation, when technology jumps from a to a^*, the overall value of the firm does not change, but its components do: the expected flow of profits increases and the value of the option to innovate drops.

Figure 3: MNC Optimal Innovation Policy with $\Delta\omega_t = 0$ for all t

(a) Capacity Utilization (b) Technology

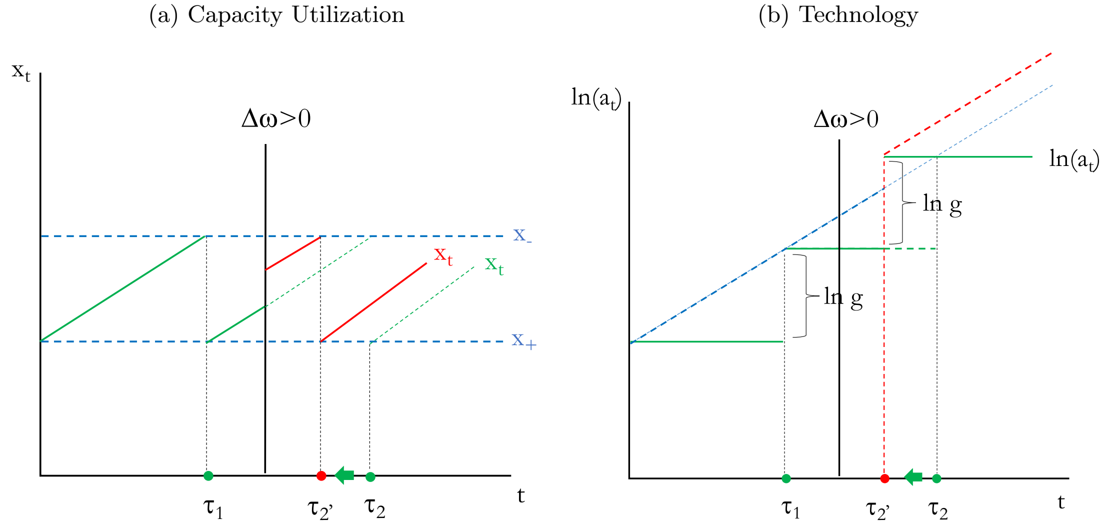


Note: The diagrams show the innovation dynamics when the realization of the firm's random demand shock $\Delta\omega = 0$ for all t. The variable $x_t = M_m^2 h/a$ describes the capacity utilization at time t, for a fixed market size M_m. x_- is the capacity utilization level that triggers a new round of innovation and x_+ is its level post innovation, as defined in Proposition 1.

The timing of innovation is endogenous. The firm waits until its total sales, given by random component h and time-invariant market size M_m, are high relative to the firm's technological capacity a. At that point, capacity utilization reaches a critical point x_-, and the technology is replaced by a superior vintage $a^* = a \cdot g$, with $g > 1$.

Figure 3 shows the *road map* that characterizes the optimal innovation policy, given by the capacity level that triggers innovation x_-, and the growth of technology $g = x_-/x_+$. Absent any shocks ($\Delta\omega_t = 0$ for all t), the firm innovates at constant intervals. The random shock affects the speed at which the firm reaches x_-. A positive realization (Figure 4) shortens the inaction interval and brings forward the next round of innovation. More generally, a history of positive sales shock realizations is associated with a higher technological capacity a, because innovation occurs more frequently. Correspondingly, negative realizations delay innovation. This endogenous timing

Figure 4: MNC Optimal Innovation Policy with $\exists! t : \Delta\omega_t \neq 0$



Note: The diagram shows the innovation dynamics with a single non-zero (positive) realization of the firm's random demand shock. The variable $x_t = M_m^2 h/a$ describes the ratio of sales to technological capacity at time t, for a fixed market size M_m. x_- is the ratio level that triggers a new round of innovation and x_+ is the ratio post innovation, as defined in Proposition 1.

of innovation implies a drop in expected productivity growth following a change in technology as found in the data (Figure 2), consistent with Fact 5 in Section 2.3. This is because innovation episodes follow *good luck* (i.e., a sequence of positive realizations of $\Delta\omega$), which in expectation reverses to its mean afterward.

Result 1 (Selection and Innovation) *Everything else equal, relative to a non-innovating firm, a firm that innovates at time t grows, in expectation, faster prior to t and at the same rate afterwards. Formally:*

$$\mathbb{E}[\ln(z_{t^-}) - \ln(z_{t-\Delta})|h_t = h^*(a_{t-1})] > \mathbb{E}[\ln(z_{t^-}) - \ln(z_{t-\Delta})|h_t < h^*(a_{t-1})]$$

and

$$\mathbb{E}[\ln(z_{t+\Delta}) - \ln(z_{t^+})|h_t = h^*(a_{t-1})] = \mathbb{E}[\ln(z_{t+\Delta}) - \ln(z_{t^+})|h_t < h^*(a_{t-1})]$$

where t^- and t^+ are the instants immediately before and after the innovation episode.

Note that an unexpected increase in market size, $\Delta M_m > 0$, has the same effect as a positive demand shock. It implies a jump in capacity utilization ($x = M^2 h/a$), which triggers innovation.

This prediction is consistent with the empirical literature on the effect of unexpected market size shocks on innovation (see, for example, Bustos, 2011).

3.2.2 Optimal Entry into Multinational Activities

The problem of the domestic firm in expression (11) is more complex, as it evaluates not only when and by how much to upgrade its technology, but also whether and when to acquire a foreign affiliate. Correspondingly, the value function of the domestic firm now includes two real options, the option to innovate and the option to open an affiliate in the foreign market.

The solution is obtained backward, starting at the point of entry into MNC activities (we note this last decision node with the subscript T). We focus on the decision of entry into MNC decoupled from the upgrade in technology. In the quantitative exercise, we set the initial conditions (a_0) so that this is optimally the case. Then, the entry decision is characterized by the following Value Matching and Smooth Pasting conditions:

1. Value matching at entry. At the time of entry, given by the level of sales $h^E(a)$, the value of the domestic firm at the time of acquiring a foreign affiliate is equal to its value as a multinational, net of the entry cost.

$$V_{dT}(a, h^E) = V_m(a, h^E) - F_M$$

2. Smooth pasting condition on the timing of entry. The level of sales that defines the time to become multinational is optimal.

$$\left.\frac{dV_{dT}(h,a)}{dh}\right|_{h=h^E} = \left.\frac{dV_m(h,a)}{dh}\right|_{h=h^E}$$

where $V_m(h,a)$, the value function on entry, is given by expression (24).

Parallel to the expression in (18), we guess the following functional form for the value function of the domestic firm in the last node before entry in terms of capacity utilization $x \equiv M_d^2 h/a$:

$$V_{dT}(a,h) = aJ_{dT}(a,x) = a\left[\frac{x^{1/2}}{\widetilde{r}} - \frac{f}{r} + H_T(a)x^{\psi}\right] \tag{25}$$

The discount factor $\widetilde{r}$ and the parameter ψ are defined by equations (19) and (20), respectively. They are estimated using the same arbitrage condition during the inaction period (where inaction occurs in both the entry and innovation margins) as in the expression (14).

Entry into multinational activity requires the payment of a sunk cost F_M. This introduces the role of economies of scale. Only the most productive firms will find it optimal to pay the entry cost and become multinationals. This is why the real option parameter $H_T(a)$ is now a function of a. Then, the solution, given the technological capacity of the firm, a, consists of the optimal time of entry $h_E(a)$ and the real option function $H_T(a)$ that satisfy the two conditions above.

We derive the optimal entry policy by replacing (25) with the entry value matching and smooth pasting conditions. The results are summarized in the following proposition:[14]

Proposition 2 (Optimal Entry) *Given the size of the domestic and multinational markets $M_d < M_m$ and a finite entry cost F_M, there is a minimum technological capacity a_E to become an MNC. Once that level of technology is reached (that is, $a \geq a^E$), the firm enters into multinational activities once the level of sales, given by h, satisfies $h \geq h_E(a)$:*

$$h_E = \frac{1}{a}\left[\frac{\widetilde{r}F_M}{M_m - M_d}\left(\frac{2\psi}{2\psi - 1}\right)\right]^2$$

Notice that, rearranging terms, the entry decision takes the form of a productivity cutoff rule as is usual in static models à la Melitz (2003) and Helpman, Melitz, and Yeaple (2004). Firms become multinational when their revenue productivity reaches z_E:

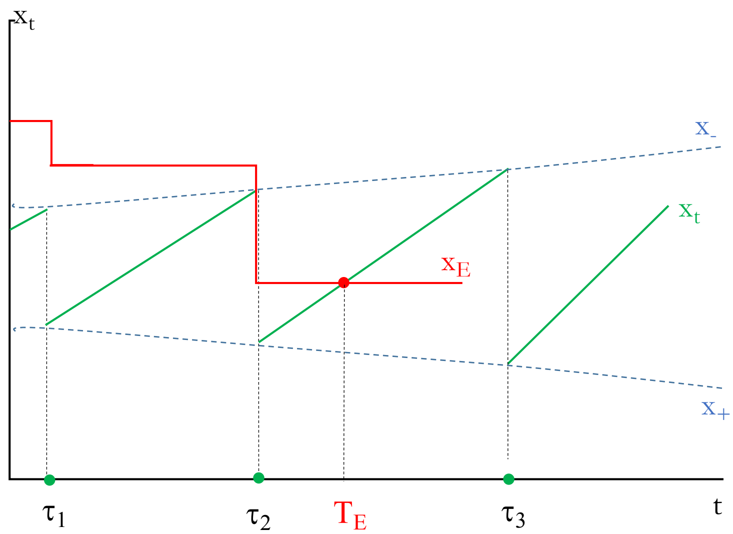
$$z_E = (ah_E(a))^{1/2} = \frac{\widetilde{r}F_M}{M_m - M_d}\left(\frac{2\psi}{2\psi - 1}\right).$$

Figure 5 represents, in red, the entry cutoff rule in Proposition 2, expressed in terms of capacity utilization $x_E(a) = M_d^2 h_E(a)/a$. In green, the figure shows the evolution of capacity utilization while the firm remains domestic, without any sales shock (ie, $\Delta\omega = 0$). As will be shown next, differently from the MNC case described in Proposition 1, the innovation policy of the domestic firm is not size-invariant. The time elapsed between innovation episodes, as well as the growth in technology at each node, is expected to increase as the firm approaches the revenue-productivity

[14] Derivations are shown in the appendix.

level that triggers the foreign acquisition. The entry into MNC activities occurs when the capacity utilization of the firm, which evolves as in the green line x_t, reaches the trigger entry level, x_E, depicted in red.

Figure 5: Optimal Entry into MNC with $\Delta\omega_t = 0$ for all t



Note: This diagram shows the optimal innovation and entry policies around the time of entry into MNC when the realization of the firm's random sales shock is zero ($\Delta\omega = 0$) for all t. $x^E(a_n)$ is the capacity utilization level that triggers entry into MNC. x_t describes capacity utilization of the firm at time t if it remains domestic. x_- is the capacity utilization level that triggers innovation, and x_+ its post innovation level. Entry into MNC occurs at T_E.

The guessed value function in (25) in the last node as a domestic firm (that is, $t \in (T-1, T)$) is verified for $H_T(a) = C_T + D_T a^{2\psi-1}$ with

$$C_T = C\left(\frac{M_m}{M_d}\right)^{2\psi} \tag{26}$$

$$D_T = \widetilde{\psi}\, F_M^{1-2\psi} \left(\frac{M_m - M_d}{M_d \widetilde{r}}\right)^{2\psi} \tag{27}$$

where C is the constant that characterizes the value of innovation option for the MNC in Equation (24) and $\widetilde{\psi} = (2\psi-1)^{2\psi-1}(2\psi)^{-2\psi}$.

Replacing, the value function in node T is given by the following expression:

$$V_{dT}(a,h) = \frac{(ah)^{\frac{1}{2}} M_d}{\widetilde{r}} - \frac{fa}{r} + C\ a\left(\frac{M_m^2 h}{a}\right)^{\psi} + \widetilde{\psi} F_M^{1-2\psi}\left(\frac{M_m - M_d}{\widetilde{r}}\right)^{2\psi}(ah)^{\psi} \tag{28}$$

The first two terms in (28) are, as usual, the discounted flow of expected profits as a domestic firm. The remaining terms correspond to two real options: the first is the option to innovate. Given that this is the last technology vintage for a domestic firm, any future innovation would affect the flow of profits once the company becomes multinational. Therefore, the option to innovate increases in M_m. Finally, the last term corresponds to the option of entering into multinational activities. The value of this option increases with the difference in market size $M_m - M_d$, and decreases in the entry cost, F_M.

Notice that technological capacity has opposite effects on the values of the two real options. The option to innovate is more valuable when the firm's technological capacity is low (relative to its level of production). Instead, the FDI option increases in technological capacity, as higher productivity brings the firm closer to opening its first foreign affiliate.

The dynamic selection effect described in Result 1 also applies to this case: A baby multinational has, in expectation, a recent history of positive sales growth, relative to non-entering firms, prior to entry and reverts to mean afterwards, qualitatively consistent with Fact 3 in Section 2.2. Figure 5 plots the deterministic productivity *roadmap* that characterizes the entry decision (i.e., $\Delta\omega_t = 0$ for all t). Although the entry condition is deterministic, positive (negative) realizations of sales shocks $\Delta\omega > 0$ ($\Delta\omega < 0$) bring forward (delay) both the frequency of innovations and the moment of entry.

The same mechanism contributes to the innovation decision of baby multinationals. Entry follows good realizations of the random sales shock, which also brings the firm closer to the next round of innovation. And, fundamentally, a now larger multinational market ($M_m > M_d$) implies a discrete increase in capacity utilization and is likely to trigger innovation, consistent with Fact 2 in Section 2.2

This intuition is summarized in the following result.

Result 2 (Selection into FDI and Innovation) *Consider domestic firms with technological capacity $a \geq a_E$. In expectation, a firm that enters into MP experiences higher sales growth prior to entry relative to a firm that remains domestic, and reverts afterwards. Formally:*

$$\mathbb{E}[\ln(z_{t^-}) - \ln(z_{t-\Delta})|h_t = h_E(a)] > \mathbb{E}[\ln(z_{t^-}) - \ln(z_{t-\Delta})|h_t < h_E(a)]$$

$$\mathbb{E}[\ln(z_{t+\Delta}) - \ln(z_{t^+})|h_t = h_E(a)] = \mathbb{E}[\ln(z_{t+\Delta}) - \ln(z_{t^+})|h_t < h_E(a)]$$

Moreover, entry into MP is likely to trigger a new round of innovation:

$$Pr[h_{t+\Delta} \geq h^*(a)|h_t = h_E(a)] > Pr[h_{t+\Delta} \geq h^*(a)|h_t < h_E(a)]$$

where t^- and t^+ are the instants immediately before the moment of entry into MP, and $h^*(a)$ is the level of market-penetration that triggers innovation.

3.3 Domestic Firm's Optimal Innovation Policy

The domestic firm innovation policy is solved backwards, starting from the firm's technological capacity, a_T, and the value function $V_{dT}(h,a)$ in (28), at the last node before entry into the MNC, described in Proposition 2. We then characterize the technology path $\{a_n\}$, the critical sales levels $\{h_n^*\}$, and the value function $\{V_{dn}(h,a)\}$ for each decision node $n = T-1, T-2, ..., 0$.

The value function and the innovation policy satisfy the same optimality conditions as the MNCs'. First, at the time of innovation, the difference in the value of the firm before and after the technological upgrade is equal to the cost of innovation (condition 15). Second, the new technological capacity is optimal (condition 16). And finally, the timing of the innovation is optimal (condition 17).

To characterize the firm's optimal policy, we first verify that the structure of the value function in this last node before entry into MNC in equation (25) is preserved during the lifetime of the domestic firm for all $n < T$

$$V_{dn}(x,a) = aJ_n(x,a) = a\left[\frac{x^{\frac{1}{2}}}{\widetilde{r}} - \frac{f}{r} + (C_n + D_n a^{2\psi-1})x^{\psi}\right] \tag{29}$$

where $x = \frac{h}{a}M_d^2$ is the capacity utilization of the domestic firm.

The solution consists of $x_{-,n}$, $x_{+,n}$, C_n and D_n that satisfy the three conditions below, the Value Matching, and the Smooth Pasting Conditions on technology level, a, and sales, h, parallel to equations (21)–(23) for the MNC case:

$$
\begin{aligned}
J_n(x_{-,n}, a_n) &= \frac{x_{-,n}}{x_{+,n}} \left[J_{n+1}\left(x_{+,n}, a_{n+1}\right) - p \right] && (30) \\
J_{n+1}\left(x_{+,n}, a_{n+1}\right) - p &= \frac{\partial J_{n+1}\left(x_{+,n}, a_{n+1}\right)}{\partial x} x_{+,n} - (2\psi - 1) D_{n+1} a_{n+1}^{2\psi-1} x_{+,n}^{\psi} && (31) \\
\frac{\partial J_n(x_{-,n}, a_n)}{\partial x} &= \frac{\partial J_{n+1}\left(x_{+,n}, a_{n+1}\right)}{\partial x} && (32)
\end{aligned}
$$

and $a_{n+1}/a_n = x_{-,n}/x_{+,n}$.

Unlike the MNC case, these conditions are affected by the level of technology a. This is because the option to expand internationally provides additional incentives to increase productivity. Then, the value function, timing, and technology growth rate change as the firm gets closer to exercising the option to become a MNC.

Consider equation (31), which is the first-order condition for the new technology capacity of the firm. Replacing with our guessed functional form in (29), it can be written as follows:

$$
\frac{x_{+,n}^{\frac{1}{2}}}{2\widetilde{r}} - \left(\frac{f}{r} + p\right) - \left[(\psi - 1) C_{n+1} - \psi D_{n+1} a_{n+1}^{2\psi-1}\right] x_{+,n}^{\psi} = 0
$$

The first two terms correspond to the classic trade-off between marginal profits and the marginal cost of increasing technology a. The term in brackets corresponds to its effect on the value of the two real options. Unlike in the MNC case, this term is no longer constant, as the option to enter into MP, governed by $D_{n+1} a_{n+1}^{2\psi-1}$, increases in the technological capacity of the firm.

As the firm approaches the cutoff for becoming multinational, the optimal policy targets a lower capacity utilization x_+ (i.e., higher a). Or, in other terms, the rate of growth of technological capacity increases as the firm becomes more productive, as depicted in Figure 5. Correspondingly, when the option of entering MP is irrelevant, either because the firm is very small ($a_n/a_E \to 0$) or because the entry into the MNC does not involve an increase in market size for the productivity of the HQ ($D_n \to 0$), the innovation policy and the value function of the domestic firm converge to the ones characterized in Proposition 1.

The following proposition, derived in the appendix, summarizes the results.

Proposition 3 (Optimal Innovation by Domestic Firms) *Given the parameters of the value function, C_T and D_T in* (26)-(27)*, and the technological capacity at the MNC entry node, a_T, the* *optimal innovation policy is determined backward by a sequence of technologies* a_n *and sales levels, represented by* h_n^* *for* $n = T-1, ..., 0$*, so that if* h *reaches* h_n^**, the firm upgrades its technology by a growth rate* $a_{n+1} = a_n \cdot g_n$*. The equilibrium sequence is given (backward) by the following expressions:*

$$h_n^* = \frac{a_{n+1}}{M_d^2}\, x_{+,n} \qquad g_n = \frac{x_{-,n}}{x_{+,n}} > 1$$

where $x_{-,n}$ *and* $x_{+,n}$ *satisfy the conditions* (30)-(32)*. This sequence is characterized by:*

(i) the technology and sales growth rates between innovation episodes increase as the firm becomes more productive: $x_{-,n+1} > x_{-,n}$ *and* $x_{+,n+1} < x_{+,n}$*;*

(ii) for young (unproductive) firms, infinitely distant from becoming a MNC, the technology and sales growth rates between innovation episodes converge to those of the MNC firm: $\lim_{T\to\infty} x_{-,0} = x_-$ *and* $\lim_{T\to\infty} x_{+,0} = x_+$*.*

As in the case of the MNC's innovation path, the sequence of technological upgrades for a firm with initial technology a_0 is deterministic. The realizations of the random sales shock determine the time elapsed between technological upgrades but not the growth rate in technology at each node.

Result 3 (History of Innovation and Entry into MP) *Everything else equal, firms that enter into multinational activities have a history of more frequent innovation than those that remain domestic. Moreover, in expectation, domestic firms innovate less often than mature MNCs.*

4 Quantitative Exercises

The results obtained in the previous section qualitatively match the stylized facts about the innovation of Baby MNCs we presented in Section 2. Now we turn to estimating the parameters of the model so that we can use it to obtain the quantitative effects of multinational expansion on productivity.

We proceed as follows: First, we present the logic behind the parameter estimation. We then simulate the effect of multinational activity on the productivity dynamics of the firm.

4.1 Parameter Estimation: Innovation

We now estimate the parameters that govern the growth dynamics of the firm. The calibrated parameters are chosen to minimize the distance between the differential growth rate (relative to non-innovating firms) estimated using simulated and actual data, weighted by the corresponding standard deviation. For this, we use the observed sales growth and the timing of the innovation episodes of Spanish firms in the sample, described in Section 2.3. In the simulations, we discretize time, which in our model is continuous, to the annual frequency of the data.

The simulated differential sales growth rate at time $\tau \in [t-1, t+2]$ is given by:

$$\mathbb{E}[\Delta \ln S_\tau | I_t = 1] - \mathbb{E}[\Delta \ln S_\tau | I_t = 0] = \frac{1}{2}\ln(g)\mathbb{I}(\tau = t) + \frac{1}{2}\left(\mathbb{E}[\Delta \ln h_\tau | I_t = 1] - \mathbb{E}[\Delta \ln h_\tau | I_t = 0]\right) \tag{33}$$

According to our model, sales dynamics around the time of innovation informs us both on the chosen upgrade of technology and on the parameters governing the exogenous random process of sales. First, the magnitude of technological upgrade g, captured by the first term in equation (33), impacts the differential growth rate at the time of innovation, t. And second, the endogenous timing of innovation, captured by the second term, informs us about the volatility of the random process h. As stated in Result 1, selection implies a positive differential sales growth *prior* to the innovation episode relative to non-innovating firms.

Figure 6 illustrates the calibration exercise and the role of selection. Both panels show the estimated sales growth differential using actual (red line) and simulated (blue line) data. In Panel (a), we allow for endogenous positive selection. Although all parameters are jointly determined, we can intuitively link the calibration of $\sigma = 0.04$ with the sales growth of 2.7% before innovation ($\tau = t - 1$), which we call the *selection effect*. The simulated data in Panel (b) is the result of a model without selection; that is, $\Delta\omega = 0$, so that the second term in equation (33) is zero prior to the innovation episode.

During the year of innovation ($\tau = t$), the differential growth in sales combines the two effects in equation (33), the replacement of technology for a superior one, and the reversal of the selection effect upon innovation. In our calibration, the instantaneous technological upgrade is 6.9% ($g = 1.07$), resulting in the observed average annual sales growth of 3.1% during the year of innovation, compared to firms that do not innovate.

Table 7: Calibrated Parameters

Panel A: Innovation and Productivity				
r	μ	σ	p	f
0.08	0.01	0.04	4e-04	0.52
Panel B: Multinational Expansion				
	M_{x0}/M_0	M_m/M_0	F/M_m	$M_{x,m}/M_{x,0}$
	0.36	1.16	1.52	0.78

Note: The annual interest rate r is pinned at 8%, and the share of exports to total sales prior to the multinational expansion is pinned to the average in the data, 36%. The rest of the parameters are obtained from minimizing the distance (weighted by the corresponding standard deviation) between β_τ estimated using actual and generated data from the following regressions. For Panel A: $\Delta \ln(Sales_{it}) = \sum_{\tau=t-1}^{t+1} \beta_\tau Inn_{i\tau} + \epsilon_{it}$. For Panel B: the distance is minimized jointly over three regressions — $\Delta \ln(Sales_{it}) = \sum_{\tau=t-1}^{t+1} \beta_\tau Entry_{i\tau} + \epsilon_{it}$, and the same specification with domestic sales and exports as the dependent variable.

Finally, the growth differential reverts to zero after innovation ($\tau > t$), since the past performance does not predict future realizations of the random shock. This is a hard constraint on the model and does not depend on the calibrated parameters. We take the fit of post-innovation growth rate as a validation of our modeling assumption.

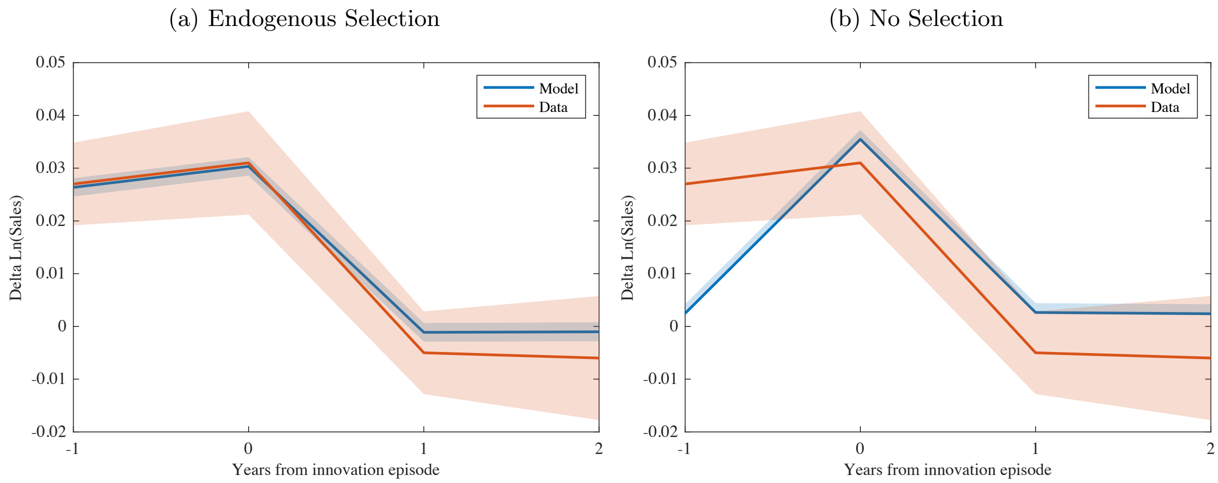
The complete set of calibrated parameters can be seen in Panel A of Table 7. r and μ do not independently affect the dynamics of the simulated firm. Then, without loss of generality, we fixed the annual interest rate to $r = 8\%$, which, given $\sigma = 0.04$, implies $\mu = 0.01$. These three parameters jointly determine $\tilde{r}$ and ψ in equations (19) and (20). Finally, the growth of technology $g = 1.07$ requires $f = 0.52$ and a positive, although very small, p.

4.2 Parameter Estimation: Multinational Expansion

Having obtained the parameters governing the sales random process and innovation, we use the information on Spanish firms' sales growth around the time of entry into multinational activities to estimate the change in market size and entry cost associated with the opening of the first foreign affiliate. The change in total sales from the model is given by:

$$\begin{aligned} \mathbb{E}[\Delta \ln S_\tau | E_t = 1] - \mathbb{E}[\Delta \ln S_\tau | E_t = 0] \quad = \quad & \Delta \ln(M_{x,\tau}) \mathbb{I}(\tau > t) + \frac{1}{2} \ln(g)\ \mathbb{I}(I_\tau = 1) + ... \\ & ... + \frac{1}{2} \left(\mathbb{E}[\Delta \ln h_\tau | E_t = 1] - \mathbb{E}[\Delta \ln h_\tau | E_t = 0] \right) \end{aligned} \quad (34)$$

Figure 6: Sales Growth and Innovation: Data and Simulation



Note: Coefficients of the OLS regression $\Delta \ln(Sales_{it}) = \sum_{\tau=t-1}^{t+1} \beta_\tau Inn_{i\tau} + \epsilon_{it}$. In both panels, the data-series corresponds to coefficients estimated on actual data from Spanish firms. In the model-series, the data used in the estimation simulates 100 firms over 10 years using parameters in Table 7. Panel (b), model-generated data imposes $\Delta\omega = 0$ (i.e., no selection).

We obtain this increase in the market size served by HQ's productivity from the difference in HQ's sales growth between firms that enter into multinational activities at t ($E_t = 1$) vs. those that do not ($E_t = 0$) for $\tau \in (t-1, t+2)$. The market size computed here *does not* correspond to the actual increase in international sales of the newly created MNC, but to the range of multinational activities that benefited from the HQ's innovation. This conceptual market size is the relevant element behind the innovation decision of the *baby multinational*; it refers to the transmission of HQ's technology within the corporation.

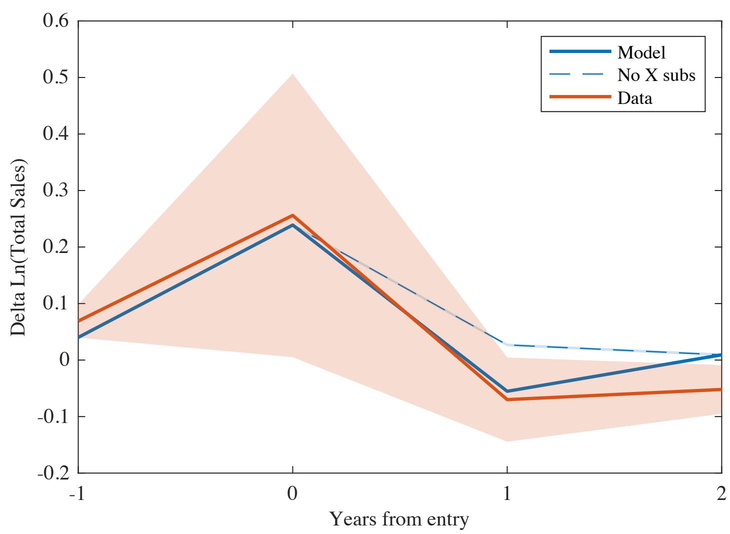
The estimated parameters in Panel B of Table 7 minimize the distance (weighted by the corresponding standard deviation) between actual and simulated moments from three regressions: differential growth of total HQ sales, domestic sales, and exports around the time of entry into multinational activities.

Selection, given by the last term in equation (34), contributes to the differential growth of sales *up* to the moment of entry. Good realizations of the productivity process lead to the endogenous decision to acquire a foreign affiliate. This selection component is driven by the parameters of the random process (μ and σ) estimated in the previous subsection, coupled with the fixed entry cost relative to the change in market size, $F/(M_m - M_0)$, which governs the entry decision.

The term $\frac{1}{2}\ln(g)\,\mathbb{I}(I_\tau = 1)$ captures the increase in sales due to technological innovation, which is more likely to occur immediately after the multi-national expansion. This post-entry technological upgrade is tailored to the new market size, which we estimate to be 16% larger ($M_m/M_0 = 1.16$). The resulting update in technology, together with the random process, results in a differential sales growth of 25.6% during the year of entry, as found in our data.

$\Delta\ln(M_{x,\tau})$ corresponds to the potential substitution of HQ exports in favor of multinational sales after the acquisition. This term may negatively affect HQ sales after the multinational expansion ($\tau > t$). We estimate that this substitution rate is 22% of the size of the export market prior to entry ($M_{x,m}/M_{x,0} = 0.78$). Without this effect, the evolution of sales at the headquarters would follow the dotted line in Figure 7. To be clear, the MNC firm may have incentives to innovate even if the number of markets supplied from the HQ drops. This is because innovation at home improves not only the efficiency of the HQ but also that of the foreign affiliate.

Figure 7: Sales Growth and Entry into MP: Data and Simulation



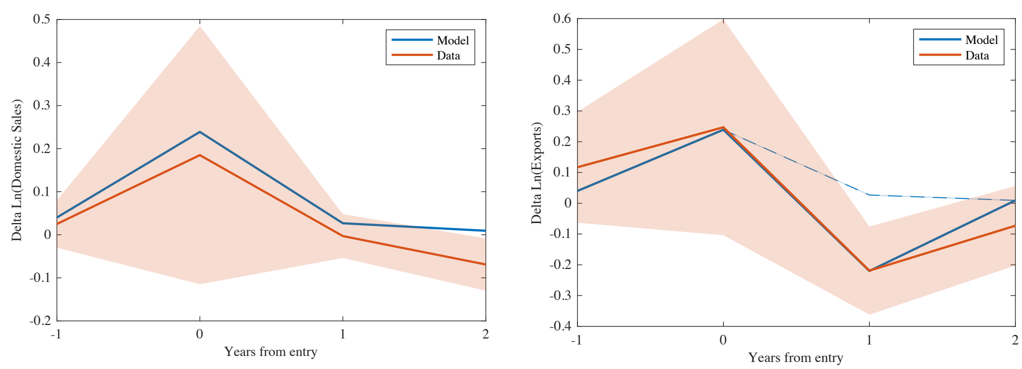
Note: All series show the coefficients of the OLS regression $\Delta\ln(Sales_{it}) = \sum_{\tau=t-1}^{t+1}\beta_\tau FDI_{i\tau} + \epsilon_{it}$ and the 95% CI. In red, the coefficients are estimated using actual data by domestic firms and HQs of BabyMNCs (with industry fixed effects). The coefficients in blue are estimated on model-generated data using parameters in Table 7. The dotted-blue series imposes no substitution between exports and foreign affiliate sales, so that $M_{x,m} = M_{x,0}$.

Figure 7 shows that the data generated by the model fits quite remarkably the differential growth rate of total HQ sales around the time of entry into multinational activities. Figure 8 shows that this success extends to the decomposition between domestic sales and exports. Panel

(a) shows the differential growth of domestic sales, which is lower before entry than total HQ sales. The model captures this pre-entry selection effect and the peak at the year of entry, falling within the confidence interval of the data throughout. Panel (b) shows the differential export growth, which is higher than for total sales prior to entry. The model fits the export profile closely, and in particular replicates the sharp reversal immediately after entry — from a large positive growth at the year of entry to a sharp decline the following year. In our model, a single productivity process and productive capacity determines all HQ sales, regardless of destination. Under this assumption, the differential evolution of exports and domestic sales around the time of entry reflects entirely the substitution of HQ exports in favor of sales by the foreign affiliate, which we calibrate at 22% of the pre-entry export market.

Figure 8: Domestic Sales and Exports: Data and Simulation

(a) Domestic Sales (b) Exports



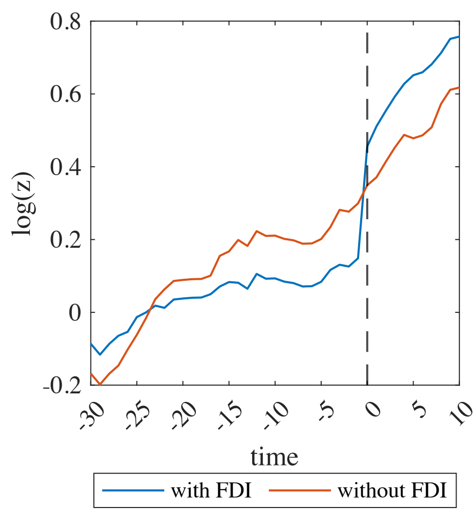
Note: All series show the coefficients of the OLS regression $\Delta \ln(Sales_{it}) = \sum_{\tau=t-1}^{t+1} \beta_\tau FDI_{i\tau} + \epsilon_{it}$ and the 95% CI. In red, the coefficients are estimated using actual data by domestic firms and HQs of BabyMNCs (with industry fixed effects). The coefficients in blue are estimated on model-generated data using parameters in Table 7. The dotted-blue series imposes no substitution between exports and foreign affiliate sales, so that $M_{x,m} = M_{x,0}$.

4.3 Effect of Multinational Expansion on HQ Productivity and Sales

Using the parameters in Table 7, we simulate the growth path of the same firm, subject to the same realization of the random sales process, under two alternative models: one where there is an option of multinational expansion, as modeled here, and another in which this option does not exist. This exercise is meant to isolate the effect of the possibility of multinational expansion on the Home unit productivity and sales, without the selection effect that endogenously drives the expansion decision.

Although the sales random component h_{it} is common to the firm in both scenarios, the resulting sales paths in Figure 10 are different. The option of multi-national expansion affects the innovation decision of the domestic firm prior to entry. The size of the technology increments and the time elapsed between innovation episodes both increase as the firm gets closer to the entry productivity cutoff. Before entry, these two forces pull in opposite directions — each larger technology jump lifts the blue line above the red, while the longer gaps between episodes allow the red line to catch up — so the two lines cross repeatedly. Multinational expansion clearly implies a higher level of productivity after entry. At that point, the technological capacity adjusts to its optimal level, which is 36% higher than in the scenario without multinational expansion. Given the decreasing returns on technology expansion (equation 4), this superior technological capacity translates into a 16% increase in the productivity of the Home unit, relative to the non-MNC counterfactual.

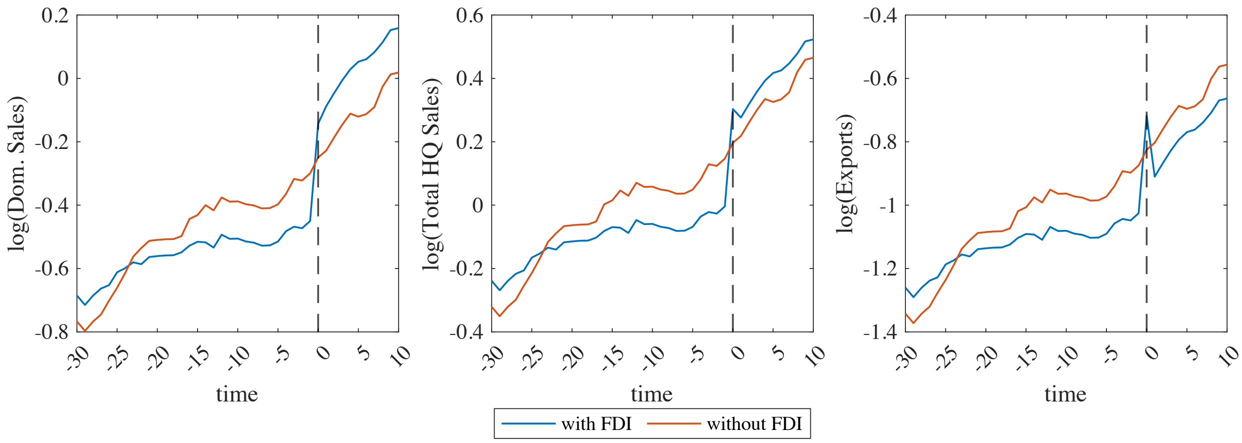
Figure 9: Effect of Multinational Expansion on Home Revenue Productivity



Note: Simulation of (log of) revenue productivity (z_t in equation 4) in the Home unit. In blue, scenario with option of multinational expansion (which occurs at $t = 0$). In red, scenario without multinational expansion. Path of random process h_t and initial conditions h_0, a_0 common to both cases.

The productivity differential translates one-to-one into domestic sales, which is 16% larger in the scenario with multinational activity. However, for overall sales by the HQ unit, the difference between the two scenarios is reduced to 7%, since HQ exports are 9% lower than in the non-FDI case. Importantly, total sales to foreign destinations do not decrease: they are now partly performed by the foreign affiliate, which also benefits from the improved technology of the multinational corporation. It is HQ exports specifically that are lower, as they are partly substituted by affiliate sales.

Figure 10: Effect of Multinational Expansion on Home Sales



Note: Simulation of (log of) domestic sales, total sales and exports by the Home unit. In blue, scenario with option of multinational expansion (which occurs at $t = 0$). In red, scenario without multinational expansion. Path of random process h_t and initial conditions h_0, a_0 common to both cases.

5 Conclusion

In this paper, we analyze the joint process of innovation and multinational activities of firms. Using panel data on Spanish manufacturing firms, with uniquely detailed information on innovation and international activities, we start by documenting some stylized facts that guide our theoretical framework. First, we show that innovation is a lumpy and disruptive process. Changes in the technology, organizational structure or inputs to production occur sporadically and imply an immediate drop in sales growth. And second, the motive for the first foreign acquisition is *horizontal*. That is, the objective of the foreign affiliate is to sell or distribute products similar to those sold in the home country, rather than to produce inputs for the headquarters.

Based on these stylized facts, we model innovation and market expansion as two complementary discrete options. We model innovation as a replacement of existing technology for a superior vintage. As long as technology in the headquarters is at least partially transmittable to the foreign affiliate, a multinational expansion provides incentives to further expand the technological capacity of the parent firm. Consequently, domestic firms with a history of more innovation are endogenously more likely to enter into multinational activities.

Through the lens of this model, we are able to disentangle the effect of multinational market expansion on the innovation of the headquarter firm independently of the positive self-selection bias. According to our calibration, the first multinational expansion implies, on average, a 16% increase in the HQ's productivity. Being a partial equilibrium model, we cannot conclude on the overall impact of multinational activities on source country productivity. However, we note that some general equilibrium mitigating forces typically important in analyzing the effect of trade on productivity are expected to have less of an impact in this case. Multinational expansion is different from exports in that the former implies relocation of production. According to our calibration, foreign acquisition also involves the substitution of 22% of the pre-entry export market in favor of multinational sales, which limits the impact of technological upgrade on local input markets. We leave this general equilibrium analysis for future research.

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A.1 Additional Tables and Figures

Table A.1: Innovation around the Time of Entry: Linear Probability Models

	Any Innovation$_{it}$ (1)	Product Innovation$_{it}$ (2)	Process Innovation$_{it}$ (3)
ΔFDI_{it+1}	0.051	0.063	0.016
	(0.066)	(0.037)	(0.066)
ΔFDI_{it}	0.162***	0.148**	0.143**
	(0.048)	(0.052)	(0.052)
ΔFDI_{it-1}	0.242***	0.262***	0.194***
	(0.081)	(0.079)	(0.073)
ΔFDI_{it-2}	0.185**	0.146**	0.105
	(0.072)	(0.057)	(0.077)
Year FE	Yes	Yes	Yes
Obs.	17991	17991	17991

Note: Estimates from linear probability models, estimating the probability of doing any innovation (product or process, column 1), product innovation (column 2), and process innovation (column 3). *Baby MNCs* are matched to domestic firms in the first year firms are in the sample through the propensity score procedure described in section 2.2. Standard errors in parentheses are clustered at the industry level. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

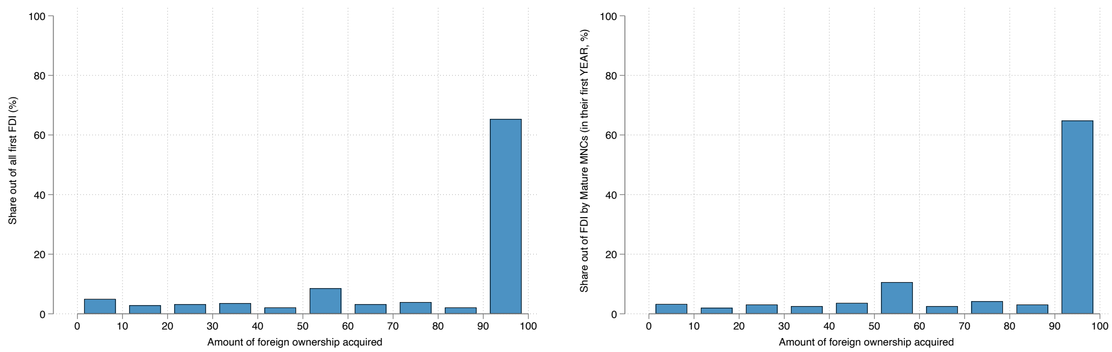
Table A.2: Destination of Foreign Investment

	All MNCs
European Union	50%
Spanish-speaking countries	15%
U.S.	8%
Asia	5%

Note: The distribution of destinations is calculated across all observations for which MNCs report a stake in a foreign affiliate of at least 50%. Firms report the home country of their main affiliated company.

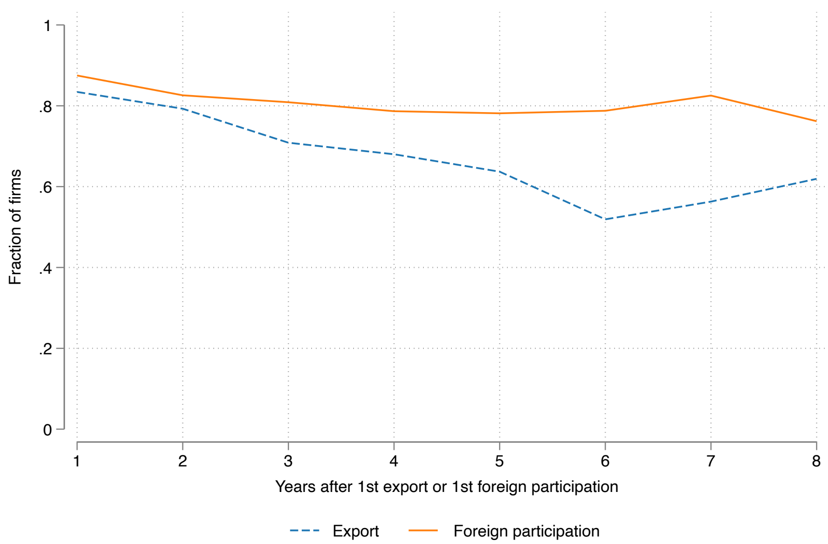
Figure A.1: Stake Control

(a) Baby MNCs (b) Mature MNCs



Note: Panel (a) plots the share of *Baby MNCs* according to their percentage stake in the first foreign investment. Panel (b) plots the share of Mature MNCs according to their percentage stake in foreign investment in the first year we observe them.

Figure A.2: Exit from International Activities



Note: Figure depicts the fraction of firms that remain Exporters or MNC in the n-year after first entering the corresponding international activity.

Table A.3: Destination of First Export

	First export
Exports to a single destination region	77%
Single destination region is European Union (E.U.)	81%
Single destination region is non-E.U. OECD	4%
Single destination region is Latin America	0%
Single destination region is Rest of the World	15%

Note: The distribution of destinations is calculated across all observations for which we observe the firm entering into export activity. Firms report the distribution of exports by destination region (European Union, OECD, Latin American, Rest of the World).

A.2 Proofs and Derivations

- **Optimal Innovation by MNCs**

The guessed value function for the MNC is $V_m(a,h) = aJ_m(x)$ with $x = M_m^2 h/a$ and

$$J_m(x) = \frac{x^{1/2}}{\widetilde{r}} - \frac{f}{r} + C_m x^{\psi}$$

with $\widetilde{r}$ and ψ defined in (19) and (20) respectively. Replacing in the conditions (21), (22) and (23) we obtain the following system of equations:

$$\frac{x_-^{1/2}}{\widetilde{r}} - \frac{f}{r} + Cx_-^{\psi} = \frac{x_-}{x_+}\left[\frac{x_+^{1/2}}{\widetilde{r}} - \left(\frac{f}{r} + p\right) + Cx_+^{\psi}\right] \tag{A.1}$$

$$\frac{x_+^{1/2}}{\widetilde{r}} - \left(\frac{f}{r} + p\right) + Cx_+^{\psi} = \frac{x_+^{1/2}}{2\widetilde{r}} + \psi C x_+^{\psi} \tag{A.2}$$

$$\frac{x_-^{1/2}}{2\widetilde{r}} + \psi C x_-^{\psi} = \frac{x_-}{x_+}\left[\frac{x_+^{1/2}}{2\widetilde{r}} + \psi C x_+^{\psi}\right] \tag{A.3}$$

Given that this problem is invariant to the level of a and h, the three unknowns, x_-, x_+ and C_m are constant, given by the parameters p, f, μ, σ and r (ψ and $\widetilde{r}$ are themselves combinations of these parameters).

There are multiple equivalent ways of solving these implicit functions. Here, we provide the solutions in terms of $g = x_-/x_+$, which is then implicitly defined as a function of the parameters of the model.

Using (A.1) and $x_- = x_+ g$, we get the following.

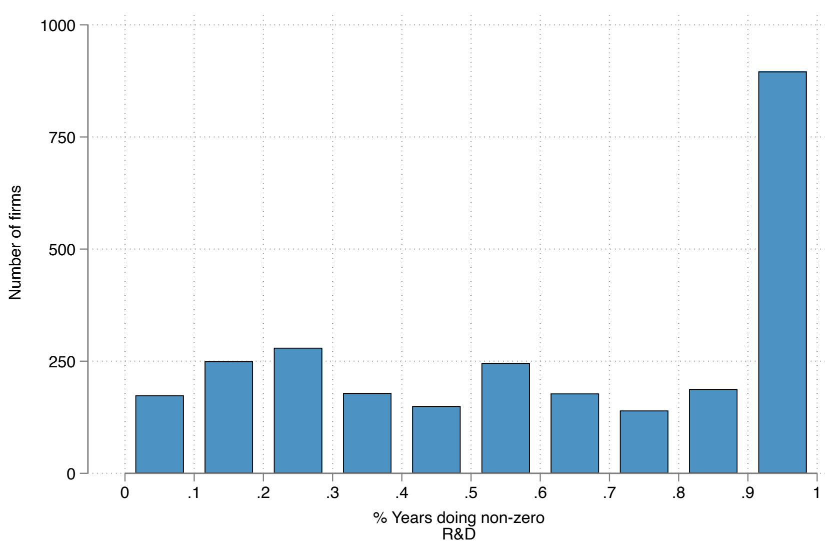
$$Cx_+^{\psi}(g^{\psi} - g) = \frac{x_+^{1/2}}{\widetilde{r}}(g - g^{1/2}) - g\left[\frac{f}{r}\left(\frac{g-1}{g}\right) + p\right]$$

Table A.4: Sales around the Time of First Export

Dep Var	$\Delta\ln(Sales_{it})$ (1)	$\Delta\ln(DomSales_{it})$ (2)
$\Delta EXPORTER_{it+1}$	0.020	0.020
	(0.015)	(0.014)
$\Delta EXPORTER_{it}$	0.122***	-0.034
	(0.035)	(0.040)
$\Delta EXPORTER_{it-1}$	0.074	0.035
	(0.021)	(0.033)
$\Delta EXPORTER_{it-2}$	0.076***	0.080**
	(0.035)	(0.035)
Obs.	6135	6120
Year FE	Yes	Yes

Note: Estimates from OLS regressions in first-differences. New Exporters are matched to Non-Exporters in the first year firms are in the sample. The matching is performed using the propensity scores on size, industry, productivity, average wage, (log) physical assets and (log) employment at the year they first enter the database. Standard errors in parentheses are clustered at the industry-level. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Figure A.3: Frequency of Doing R&D Activities



Note: Note: Figures based on the entire ESEE database from 1990 to 2010. Figures display the number of firms that report non-zero R&D expenditures in each fraction of years as indicated on the X-axis.

Replacement in (A.3), it leads to the solution for x_+ and x_- in terms of g:

$$\begin{aligned} \frac{x_+^{1/2}}{2\widetilde{r}} &= \frac{\psi}{2\psi-1}\frac{g^{1/2}}{g^{1/2}-1}\left[\frac{f}{r}\left(\frac{g-1}{g}\right)+p\right] \\ \frac{x_-^{1/2}}{2\widetilde{r}} &= \frac{\psi}{2\psi-1}\frac{g}{g^{1/2}-1}\left[\frac{f}{r}\left(\frac{g-1}{g}\right)+p\right] \end{aligned}$$

From the first-order condition of the new technology in (A.2), we get that:

$$C = \frac{x_+^{-\psi}}{\psi-1}\left[\frac{x_+^{1/2}}{2\widetilde{r}} - \left(\frac{f}{r}+p\right)\right]$$

We get analogous expression using x_-, when combining (A.1) and (A.2) and substituting for the RHS of equation (A.3):

$$C = \frac{x_-^{-\psi}}{\psi-1}\left[\frac{x_-^{1/2}}{2\widetilde{r}} - \frac{f}{r}\right]$$

Equalizing the two expressions and using $x_- = g\ x_+$, we get an alternative expression for x_+:

$$\frac{x_+^{1/2}}{2\widetilde{r}} = \frac{g^{\psi}}{g^{\psi}-g^{1/2}}\left[\frac{f}{r}\left(\frac{g^{\psi}-1}{g^{\psi}}\right)+p\right]$$

Finally, equalizing the alternative expressions for x_+ leads to the following implicit function for the growth rate:

$$\frac{g^{\psi}}{g^{\psi}-g^{1/2}}\left[p+\frac{f}{r}\left(\frac{g^{\psi}-1}{g^{\psi}}\right)\right] = \frac{\psi}{2\psi-1}\frac{g^{1/2}}{g^{1/2}-1}\left[p+\frac{f}{r}\left(\frac{g-1}{g}\right)\right]$$

- **Result 1: Selection and Innovation**

Consider ex-ante identical during the interval $[t-\tau, t+\tau]$ (i.e., same $h_{t-\tau}$ and $a_{t-\tau}$), some of which make a single innovation during the interval, at time t, while the rest do not innovate in the entire interval. Let $I_t = \{0, 1\}$ be an indicator variable equal to 1 if the firm innovates at time t, and 0 otherwise.

Following the results in Section 3.1, the differential growth of the HQ's sales for the two groups of firms can be expressed as follows:

$$\mathbb{E}[\Delta_j \ln S_t | I_t = 1] - \mathbb{E}[\Delta_j \ln S_t | I_t = 0] = \frac{1}{2}\left(\mathbb{E}[\Delta_j \ln(h_t)|I_t = 1] - \mathbb{E}[\Delta_j \ln(h_t)|I_t = 0]\right) \qquad \text{(A.4)}$$

where $j = L, F$ corresponds to lag and forward variations defined as follows: $\Delta_L \ln x_t \equiv \ln x_{t^-} - \ln x_{t-\tau}$ and $\Delta_F \ln x_t \equiv \ln x_{t+\tau} - \ln x_{t^+}$. t^- and t^+ represent the moments immediately before and after innovation, so that these expressions for sales variations do not incorporate the technology replacement that occurs at time t.

Given the process of h, described in equation (6):

$$\Delta_j \ln(h_t) = (\mu - 0.5\ \sigma^2)\tau + \sigma\ \Delta_\tau \omega$$

Then, in expectation, the differential growth rate of h captures endogenous selection into innovation. Those that were *lucky* reach the trigger sales level, $h^*(a_{t-\tau})$, while those that were *unlucky* maintain their technology constant.

$$\begin{aligned}
\mathbb{E}[\Delta_L \ln(h_t)|I_t = 1] - \mathbb{E}[\Delta_L \ln(h_t)|I_t = 0] &= \sigma \left(\mathbb{E}\left[\Delta_\tau\omega | \Delta_\tau\omega \geq \frac{\kappa(\tau)}{\sigma}\right] - \mathbb{E}\left[\Delta_\tau\omega | \Delta_\tau\omega < \frac{\kappa(\tau)}{\sigma}\right]\right) \\
&= \sigma\sqrt{\tau} \cdot \frac{\phi(a)}{\Phi(a)\,(1-\Phi(a))} > 0
\end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal pdf and cdf, $a = \kappa(\tau)/(\sqrt{\tau}\,\sigma)$, and $\kappa(\tau) = \ln\left(\frac{h^*(a_{t-\tau})}{h_{t-\tau}}\right) - (\mu - 0.5\sigma^2)\tau$ is the size of the truncation. The expression $\phi(a)/(\Phi(a)(1-\Phi(a)))$ combines the conditional expectations from both groups: $E[Z|Z \geq a] = \phi(a)/(1-\Phi(a))$ for innovators and $-E[Z|Z < a] = \phi(a)/\Phi(a)$ for non-innovators, with $Z \sim N(0,1)$.

On the other hand, since future realizations of $\Delta_\tau\omega$ are independent of past history, innovation at time t does not predict a differential path in the random sales process between $(t^+, t+\tau)$

$$\mathbb{E}[\Delta_F \ln(h_t)|I_t = 1] - \mathbb{E}[\Delta_F \ln(h_t)|I_t = 0] = 0$$

- **Optimal Entry into MNC**

The results here, as well as the numerical exercise, are based on the case of entry into MNC decoupled from innovation. Alternatively, the firm innovates at the same instant it enters into multinational activities. Although we speculate that there is no conceptual difference between the two cases, the algebra is considerably more complicated in the latter case: the expression for the value of the option to enter is not multiplicatively separable on a. We therefore present here the algebra consistent with entry decoupled from innovation, present the conditions under which this is the case, and check that these these conditions are met in the numerical simulation.

From the optimal entry decision (decoupled from innovation), we get the following value function at node T (the last node as a domestic firm):

$$V_{dT}(a,h) = aJ_{dT}(x,a) = a\ \left[\frac{x^{1/2}}{\widetilde{r}} - \frac{f}{r} + (C_T + D_T a^{2\psi-1})x^\psi\right]$$

where:

$$\begin{aligned}
C_T &= C\left(\frac{M_m}{M_d}\right)^{2\psi} \\
D_T &= \left(\frac{F_M}{2\psi - 1}\right)^{-(2\psi-1)}\left(\frac{M_m - M_d}{2\psi\ M_d\widetilde{r}}\right)^{2\psi}
\end{aligned}$$

The entry decision in the domestic node can be characterized by a trigger that depends on a through the state variable $(ha)^{1/2}$. There exists a minimum capacity a_E such that, for $a \geq a_E$, entry occurs within that node when

$$\sqrt{ha} \geq z_E,$$

where $z_E > 0$ is a constant that depends only on the primitives M_d, M_m, F_M, $\widetilde{r}$, and ψ. Given a, this defines an entry sales trigger

$$h_E(a) = \frac{z_E^2}{a}.$$

Because a is fixed within the node, entry is then determined by the evolution of h alone: when $h \geq h_E(a)$, the firm enters multinational status.

In terms of domestic capacity utilization, this entry boundary is

$$x_E(a) \equiv \frac{M_d^2 h_E(a)}{a} = \frac{M_d^2 z_E^2}{a^2}.$$

The threshold a_E is defined by the domestic innovation boundary of the last domestic node:

$$x_E(a_E) = x_{-,T},$$

where $x_{-,T}$ is the last-node domestic innovation trigger. This makes a_E the smallest capacity at which entry occurs in the last domestic node before domestic innovation would have been triggered.

More generally, the admissible values of a in node T are bounded. The minimum such value is a_E, because if $a < a_E$ then $x_E(a) > x_{-,T}$ and the firm would innovate in node T before entering. The maximum such value is an epsilon below the one that satisfies

$$x_E(a/g_{T-1}) = x_{-,T-1},$$

because if a were any larger the firm would enter in node $T-1$ instead of reaching node T.

Entry decoupled from innovation requires the post-entry MNC utilization to remain below the MNC innovation threshold:

$$\left(\frac{M_m}{M_d}\right)^2 x_E(a) < x_{-,MNC}.$$

Because $x_E(a)$ decreases in a, define a_{EN} as the minimum capacity in node T that satisfies this equality:

$$x_E(a_{EN}) = x_{-,MNC}\left(\frac{M_d}{M_m}\right)^2.$$

Entry in node T is decoupled from innovation for all $a \geq a_{EN}$. Since $\left(\frac{M_m}{M_d}\right)^2 x_{-,T} > x_{-,T} > x_{-,MNC}$, then $a_{EN} > a_E$ and the decoupled regime is a strict subset of the entry range in node T.

These conditions are not a simple closed-form inequality in primitive parameters because $x_{-,T}$ depends on the domestic value function in the last node. In the simmulation we check numerically that these conditions are satisfied.

- **Optimal Innovation by the Domestic Firm**

The functional form of the value function at node T is preserved going backwards for $n = T -$

$1, T-2, ..., 0: V_{d,n}(a,h) = a\ J_{d,n}(x,a)$ with $x = M_d^2 h/a$ and

$$J_{d,n}(x,a) = \frac{x^{1/2}}{\widetilde{r}} - \frac{f}{r} + [C_n + D_n a^{2\psi-1}]x^{\psi}$$

with $\widetilde{r}$ and ψ defined in (19) and (20) respectively.

Knowing C_{n+1}, D_{n+1} and a_{n+1}, we replace the guessed value function in conditions (30), (31) and (32), to solve for the optimal innovation policy ($x_{-,n}$, $x_{+,n}$ and a_n) and the constants of the value function (C_n and D_n) that satisfy the following system of equations:

$$\frac{x_{-,n}^{1/2}}{\widetilde{r}} - \frac{f}{r} + \left(C_n + D_n a_n^{2\psi-1}\right) x_{-,n}^{\psi} = \frac{x_{-,n}}{x_{+,n}} \left[\frac{x_{+,n}^{1/2}}{\widetilde{r}} - \left(\frac{f}{r} + p\right) + (C_{n+1} + D_{n+1} a_{n+1}^{2\psi-1}) x_{+,n}^{\psi}\right] \tag{A.5}$$

$$\frac{x_{+,n}^{1/2}}{2\widetilde{r}} + \left(\psi C_{n+1} - (\psi-1) D_{n+1} a_{n+1}^{2\psi-1}\right) x_{+,n}^{\psi} = \frac{x_{+,n}^{1/2}}{\widetilde{r}} - \left(\frac{f}{r} + p\right) + \left(C_{n+1} + D_{n+1} a_{n+1}^{2\psi-1}\right) x_{+,n}^{\psi} \tag{A.6}$$

$$\frac{x_{-,n}^{1/2}}{2\widetilde{r}} + \psi \left(C_n + D_n a_n^{2\psi-1}\right) x_{-,n}^{\psi} = \frac{x_{-,n}}{x_{+,n}} \left[\frac{x_{+,n}^{1/2}}{2\widetilde{r}} + \psi \left(C_{n+1} + D_{n+1} a_{n+1}^{2\psi-1}\right) x_{+,n}^{\psi}\right] \tag{A.7}$$

$$a_n = \frac{x_{+,n}}{x_{-,n}}\ a_{n+1} \tag{A.8}$$

Without the option to enter MNC (i.e., $D_n = 0$), this system of equation is independent of a and the solution is the same as for MNC firms, described in Proposition 1. However, for $D_n > 0$, this problem is not time-invariant, as technology a_n changes in every innovation episode n.

We solve backwards for the optimal policy and value function in each node $n = T-1, ..., 0$, knowing C_{n+1}, D_{n+t} and a_{n+1}. We proceed as for the MNC case. First, we obtain expressions for $x_{-,n}$ and $x_{+,n}$ in terms of g_n, analogous to x_- and x_+ for MNCs. In contrast to the MNC case, the domestic growth rate of technology is not constant.

$$\frac{x_{+,n}^{1/2}}{\widetilde{r}} = \frac{2\psi}{2\psi-1} \frac{g_n^{1/2}}{g_n^{1/2}-1} \left[\frac{f}{r}\left(\frac{g_n-1}{g_n}\right) + p\right] \tag{A.9}$$

$$\frac{x_{-,n}^{1/2}}{\widetilde{r}} = \frac{2\psi}{2\psi-1} \frac{g_n}{g_n^{1/2}-1} \left[\frac{f}{r}\left(\frac{g_n-1}{g_n}\right) + p\right] \tag{A.10}$$

The growth rate is then obtained from the first-order condition (A.6) for known a_{n+1} and parameters C_{n+1} and D_{n+1}. That is, for known a_{n+1} and parameters C_{n+1} and D_{n+1}, we solve for g_n, then $a_n = a_{n+1}/g_n$ and $h_n(a_n) = x_{-,n}\ a_n/M_d^2$.

We now proceed to obtain C_n and D_n. Combining (A.6) and (A.7), we get the following.

$$\frac{x_{+,n}^{1/2}}{\widetilde{r}} - \left(\frac{f}{r} + p\right) + \left(C_{n+1} + D_{n+1} a_{n+1}^{2\psi-1}\right) x_{+,n}^{\psi} = \frac{1}{g_n}\left[\frac{x_{-,n}^{1/2}}{2\widetilde{r}} + \psi\left(C_n + D_n a_n^{2\psi-1}\right) x_{-,n}^{\psi}\right] - (2\psi-1) D_{n+1} a_{n+1}^{2\psi-1} x_{+,n}^{\psi}$$

Replacing into the value matching condition (A.5):

$$\frac{x_{-,n}^{-\psi}}{\psi-1}\left[\frac{x_{-,n}^{1/2}}{2\widetilde{r}}-\frac{f}{r}\right]=C_n+D_n a_n^{2\psi-1}-\frac{(2\psi-1)}{\psi-1}[D_{n+1}g_n^{\psi}]a_n^{2\psi-1} \tag{A.11}$$

We define $\mathcal{C}_{+,n}$ and $\mathcal{C}_{-,n}$:

$$\mathcal{C}_{+,n}\equiv\frac{x_{+,n}^{-\psi}}{\psi-1}\left[\frac{x_{+,n}^{1/2}}{2\widetilde{r}}-\left(\frac{f}{r}+p\right)\right]\qquad\mathcal{C}_{-,n}\equiv\frac{x_{-,n}^{-\psi}}{\psi-1}\left[\frac{x_{-,n}^{1/2}}{2\widetilde{r}}-\frac{f}{r}\right]$$

Notice that, without the option of entering foreign markets, these two expressions are time-invariant and equal to $\mathcal{C}_+=\mathcal{C}_-=C$, the constant in the MNC value function in (24)

Given (A.6) and (A.11), $\mathcal{C}_{-,n}$ and $\mathcal{C}_{+,n}$ satisfy the following:

$$\mathcal{C}_{-,n}=C_n-\frac{\psi}{\psi-1}D_{n+1}a_{n+1}^{2\psi-1}g_n^{-\psi+1}\qquad \textit{and}\qquad \mathcal{C}_{+,n}=C_{n+1}-\frac{\psi}{\psi-1}D_{n+1}a_{n+1}^{2\psi-1}$$

It follows that $\mathcal{C}_{+,n}=\mathcal{C}_{-,n+1}$ and:

$$C_n=\mathcal{C}_{-,n}+\left(C_{n+1}-\mathcal{C}_{-,n+1}\right)g_n^{-(\psi-1)}=\mathcal{C}_{-,n}+\left(C_T-\mathcal{C}_{-,T}\right)\left[\prod_{\tau=n}^{T-1}g_\tau^{-(\psi-1)}\right]$$

Finally, given the evolution of C_n, the guess value function is confirmed when the constant in the option value of becoming an MNC evolves as follows:

$$D_n=D_{n+1}g_n^{\psi}=D_T\left[\prod_{\tau=n}^{T-1}g_\tau^{\psi}\right]$$

Characterization of the innovation policy for the domestic firm

$\psi>1$ and $g_n>1$ for all n, so this is a contraction that converges to $C_n=\mathcal{C}_{-,n}=\mathcal{C}$ for $T-n\to\infty$. The optimal technological upgrade at $n=T-1$ is the one that maximizes the continuation value of the firm, $V_{dT}(a,h)$. The first-order condition in (31) is:

$$\frac{x_{+,n}^{1/2}}{2\widetilde{r}}-\left[(\psi-1)C_T-\psi D_T a_T^{2\psi-1}\right]x_{+,n}^{\psi}-\left(\frac{f}{r}+p\right)=0$$

Replacing with C_T:

$$\frac{x_{+,n}^{1/2}}{2\widetilde{r}}-\left[(\psi-1)C-\psi D_T\left(\frac{M_d}{M_m}\right)^{2\psi}a_{n+1}^{2\psi-1}\right]x_{+,n}^{\psi}-\left(\frac{f}{r}+p\right)=0$$

where $x_+=\frac{M_m^2 h}{a}$ is the optimal post-innovation capacity utilization, measured using MNC market size M_m. C is the MNC constant of integration in the option to innovate, which is given that $C=\frac{x_+^{-\psi}}{\psi-1}[\frac{x_+^{1/2}}{2\widetilde{r}}-\left(\frac{f}{r}+p\right)]$ for the MNC post-innovation x_+. If $D_T=0$, this FOC is satisfied for

$x_{+,n} = x_+$. For $D_T > 0$, $x_{+,n} < x_+$.

Given the expressions (A.9) and (A.10), $x_{+,n} < x_+$ implies $g_n > g$ and $x_{-,n} > x_-$. The same logic applies going backward. That is, the growth rate for domestic firms is larger than for the MNC, but so is the difference between $x_{+,n}$ and $x_{-,n}$: the optimal increase in sales between innovation episodes is larger, and therefore the time elapsed between them is in expectation longer.