Financial factors, firm size and potential

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Motivation

- Extensive focus on financial factors as a central propagation mechanism in the last few years
- Recent paper by **?** casts doubt on the existence of the financial accelerator mechanism
- Many aspects of this channel are still unresolved...
  - How important are financial frictions in affecting individual firm outcomes?
  - Do these effects show up in the aggregate?
  - What are good proxies for financial factors?

Our contribution

Empirical

1. Financial factors matter for firm cyclicality
2. This effect is orthogonal to the size effect documented in the literature
3. Ex-ante heterogeneity helps explaining differences between constrained and unconstrained firms

Theoretical (still work in progress)

1. Develop a model with financial frictions where firms differ both ex-ante and ex-post
2. This can account for the joint relevance of size and financial factors in determining firm cyclicality

Literature

- **Financial accelerator (empirical):** Gertler and Gilchrist (1994); Kashyap et al. (1994); Bernanke and Gertler (1995); Ottonello and Winberry (2018)
- **Financial accelerator (theoretical):** Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke et al. (1999); Khan and Thomas (2013)
- **Firm size:** Moscarini and Postel-Vinay (2012); Chari et al. (2013); Kudlyak and Sanchez (2017); **?**); Pugsley et al. (2018)

Data

- Use *Informação Empresarial Simplificada* data on the universe of Portuguese firms between 2006 and 2017
- Matched with Bank of Portugal credit register that records individual bank relationships and respective credit situations
- This data set is ideal for an analysis of financial constraints:
  - Data on potential credit: Banks have to report potential credit of their customers (credit lines, credit cards, etc.)
  - Very detailed: Any (potential) loan amounting to 50 Euros or more is recorded in the credit register

Data pre-processing

- As balance sheet data is reported at the end of each year, we keep only the credit information at the end of the year
- We focus only on firms which are in business at the time of reporting
- We consider only firms which are privately or publicly held (i.e. no cooperatives, public entities)
- We keep only firms with more than 5 years of reporting
- We drop micro firms, keeping only those with more than €10,000 of total credit
- The final data set comprises 176,234 firms

Results: Constrained firms are more cyclical

Run the following regressions, in the spirit of **?**:

$$g_{i,t} = \Delta GDP_t + \sum_{j \in \mathcal{J}} (\alpha_j + \beta_j \Delta GDP_t) \mathbf{1}_{i \in \mathcal{S}_t^{(j)}} + (\zeta + \eta \Delta GDP_t) \mathit{fin\_health}_{i,t}$$
$$+ \sum_{l \in \mathcal{L}} (\gamma_l + \delta_l \Delta GDP_t) \mathbf{1}_{i \in \mathcal{L}} + \epsilon_{i,t}$$

- $g_{i,t}$ is the year-on-year log change in turnover or employees
- the set $\mathcal{S}_t^{(j)}$ is a jth size group, e.g. all firms above the 90th but below the 99th percentile
- *fin_health* refers to the variable measuring the strength of financial constraints
- $\mathcal{L}$ is a set of industry dummies

Financial health variables

We define our financial health variables as follows:

- Leverage $= \frac{\text{Total debt}}{\text{Total assets}}$
- Liquidity ratio $= \frac{\text{Cash}}{\text{Liabilities}}$
- Dividends $=$ Profits $-$ Retained Earnings
- Constrained $= \mathbf{1}_{\text{Potential Credit}=0}$
- Potential Constrained $= \mathbf{1}_{\text{Total Credit}_{t+1} > \text{Total Credit}_t + \text{Potential Credit}_t}$
- Constrained continuous $= \frac{\text{Potential Credit + Cash}}{\text{Liabilities}}$

Cyclicality of Turnover

	Turnover growth			
	(1)	(2)	(3)	(4)
[90,99] × GDP growth	0.000	-0.000	-0.002*	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
[99,99.5] × GDP growth	-0.011***	-0.011***	-0.014***	-0.011***
	(0.002)	(0.002)	(0.003)	(0.002)
[99.5,100] × GDP growth	-0.014***	-0.014***	-0.019***	-0.014***
	(0.003)	(0.003)	(0.004)	(0.003)
leverage × GDP growth		0.009***		
		(0.001)		
dividends × GDP growth			0.002***	
			(0.000)	
liquidity ratio × GDP growth				-0.002*
				(0.001)
Observations	1,323,660	1,323,535	737,079	1,323,535
R-squared	0.030	0.032	0.034	0.030
Industry FE	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Sales

Cyclicality of Turnover

	Turnover growth			
	(1)	(2)	(3)	(4)
GDP growth	2.505***	2.428***	2.362***	2.518***
	(0.023)	(0.026)	(0.030)	(0.023)
[90,99] $\times$ GDP growth	-0.058	-0.004	-0.005	-0.058
	(0.105)	(0.105)	(0.105)	(0.105)
[99,99.5] $\times$ GDP growth	-0.705**	-0.641**	-0.624**	-0.724**
	(0.288)	(0.289)	(0.289)	(0.291)
[99.5,100] $\times$ GDP growth	-1.637***	-1.560***	-1.546***	-1.659***
	(0.290)	(0.291)	(0.291)	(0.291)
Constrained $\times$ GDP growth		0.265***		
		(0.053)		
Const + potential const $\times$ GDP growth			0.306***	
			(0.047)	
Constrained continuous $\times$ GDP growth				-0.101***
				(0.025)
Observations	1,368,299	1,368,299	1,368,299	1,366,750
R-squared	0.014	0.014	0.014	0.013
Industry FE	Yes	Yes	Yes	Yes
Industry FE $\times$ GDP growth	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Sales

Cyclicality of Employment

	Employees growth			
	(1)	(2)	(3)	(4)
[90,99] × GDP growth	-0.000	-0.001	-0.002***	-0.001
	(0.000)	(0.000)	(0.001)	(0.000)
[99,99.5] × GDP growth	-0.001	-0.001	-0.004**	-0.001
	(0.001)	(0.001)	(0.002)	(0.001)
[99.5,100] × GDP growth	-0.003**	-0.003**	-0.007***	-0.003**
	(0.001)	(0.001)	(0.002)	(0.001)
leverage × GDP growth		0.006***		
		(0.000)		
dividends × GDP growth			0.001***	
			(0.000)	
liquidity ratio × GDP growth				-0.003***
				(0.001)
Observations	1,287,296	1,287,123	716,046	1,287,123
R-squared	0.024	0.025	0.034	0.024
Industry FE	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Cyclicality of Employment

	Employees growth			
	(1)	(2)	(3)	(4)
[90,99] × GDP growth	-0.000	-0.000	-0.001	-0.001
	(0.000)	(0.000)	(0.000)	(0.000)
[99,99.5] × GDP growth	-0.001	-0.001	-0.001	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)
[99.5,100] × GDP growth	-0.003**	-0.003**	-0.003**	-0.003**
	(0.001)	(0.001)	(0.001)	(0.001)
Constrained × GDP growth		-0.000		
		(0.000)		
Const + potential const × GDP growth			0.001**	
			(0.000)	
Constrained continuous × GDP growth				-0.001***
				(0.000)
Observations	1,287,296	1,287,296	1,287,296	1,286,004
R-squared	0.024	0.024	0.025	0.024
Industry FE	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Discussion

We find that:

- Financial frictions matter for firm cyclicality
- E.g. the interaction coefficient of constrained firms and GDP growth is positive and significant
- Confirmation of a broader financial accelerator story

and, simultaneously, that

- Size matters also for firm cyclicality
- E.g. larger firms are less cyclical
- But this is orthogonal to the financial health!
- Confirmation of a non-financial story, too

Question: Is size a good proxy for constrained firms?

Discussion

We find that:

- Financial frictions matter for firm cyclicality
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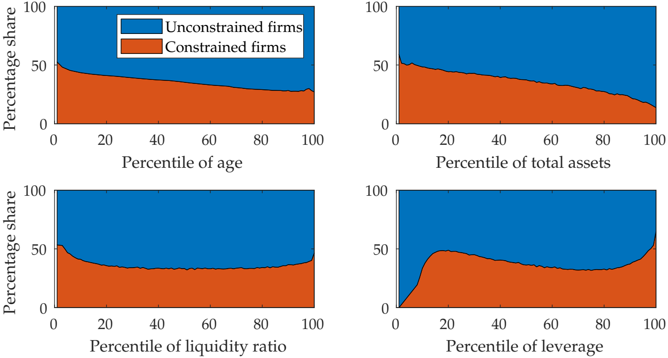
Question: Is size a good proxy for constrained firms?

Constrained firms and proxies

- What is the correlation between being constrained and firm level variables such as size?



- There are constrained and unconstrained firms in any bin
- And the correlation between the variables and constraints is as we would expect...

Linear probability model

... but this correlation is not very strong. For example, a two standard deviation increase in assets (going from the 5th to the 95th percentile) decreases the probability of being constrained only by ten percent!

	Constrained binary			
	(1)	(2)	(3)	(4)
Age	-0.05***			
	(0.000)			
Total assets		-0.05***		
		(0.001)		
Leverage			0.03***	
			(0.000)	
Liquidity ratio				0.01***
				(0.000)
Constant	0.36***	0.36***	0.36***	0.36***
Observations	1,765,288	1,765,288	1,764,947	1,764,947
R-squared	0.011	0.000	0.000	0.000

Notes: Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Discussion

- How to make sense of weak correlation between size and constrained?
  - Financial frictions model a la Khan and Thomas (2013) predicts strong correlation between the two
- Could ex-ante conditions break this correlation?
  - Small firms may be unconstrained as they already reached their potential...
  - ...while large firms may still be growing and may still be constrained
- Different potential would create a dispersion of unconstrained firms across the entire firm size distribution...
- ..although, is it reasonable to assume ex-ante heterogeneity?

Standard deviation and autocorrelation - full sample

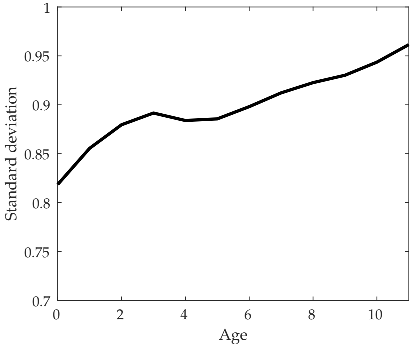


Figure: Standard deviation

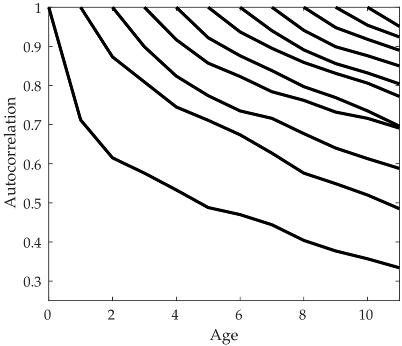


Figure: Autocorrelation

Constrained vs unconstrained

Statistical model

- In order to get a first understanding of this we estimate the Pugsley et al. (2018) model on both constrained and unconstrained firms

$$\underbrace{\ln n_{i,a}}_{\text{log employment}} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante component}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post component}}$$

where

$$\begin{array}{llllr} u_{i,a} = & \rho_u u_{i,a-1} + \theta_i, & u_{i,-1} \sim iid\left(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}\right), & \theta_i \sim iid\left(\mu_\theta, \sigma^2_\theta\right), & |\rho_u| \leq 1 \\ v_{i,a} = & \rho_v v_{i,a-1}, & v_{i,-1} \sim iid\left(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}\right), & & |\rho_v| \leq 1 \\ w_{i,a} = & \rho_w w_{i,a-1} + \varepsilon_{i,a}, & w_{i,-1} = 0, & \varepsilon_{i,a} \sim iid\left(0, \sigma^2_\varepsilon\right), & |\rho_w| \leq 1 \\ z_{i,a} \sim & iid\left(0, \sigma^2_z\right) & & & \end{array}$$

Autocovariance

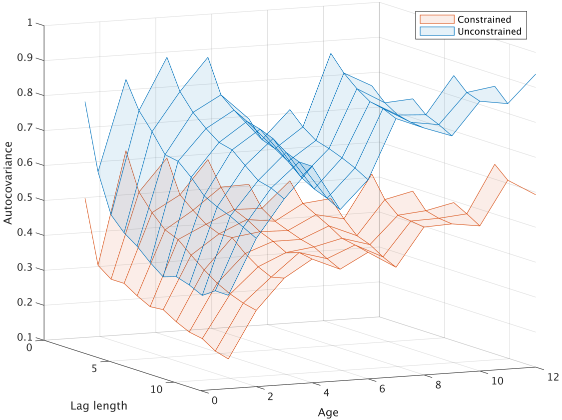
- Use autocovariance to estimate ex-ante and ex-post conditions importance
- Estimate autocovariance for two groups of firms:
  - Constrained: When a firm has potential credit equal to zero at age a-j
  - Unconstrained: Firms that have potential credit available at age a-j

$$Cov[\ln n_{i,a}, \ln n_{i,a-j}] = \underbrace{\left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right)\sigma_\theta^2 + \rho_u^{2(a+1)-j}\sigma_{\hat{u}}^2 + \rho_v^{2(a+1)-j}\sigma_{\hat{v}}^2}_{\text{Ex-ante component}}$$

$$+ \underbrace{\sigma_\epsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \sigma_z^2 \mathbf{1}_{j=0}}_{\text{Ex-post component}}$$

Derivation

Autocovariance constrained vs unconstrained - data



Model parameters

	ρ_u	ρ_v	ρ_w	σ_θ	σ_u	σ_v	σ_ϵ	σ_z
Constrained	0.485	0.760	0.707	0.339	0.009	0.671	0.287	0.147
Unconstrained	0.447	0.694	0.780	0.452	0.760	0.841	0.305	0.110

- We minimize the sum of squared deviations between the empirical autocovariance and the model autocovariance
- Model calibrated to log employment, after controlling for birth year and industry fixed effects
- Long-run steady state level of employment given by

$$\ln n_i^* = \frac{\theta_i}{1 - \rho_u}$$

- For same initial shock, constrained firms will reach a higher steady state employment due to higher ρ_u

Autocovariance: model fit

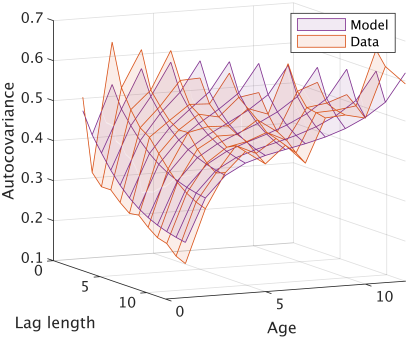


Figure: Constrained firms

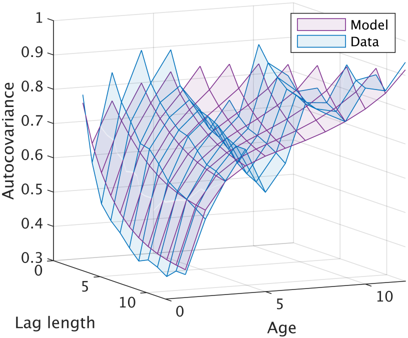
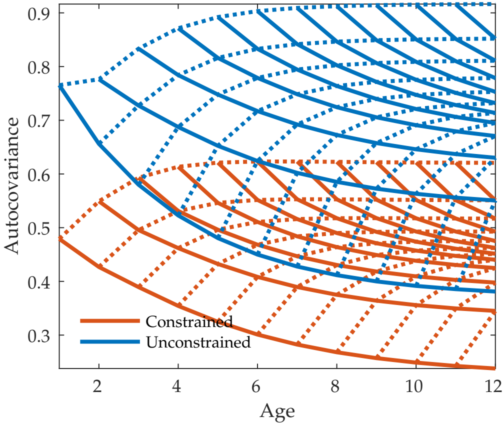
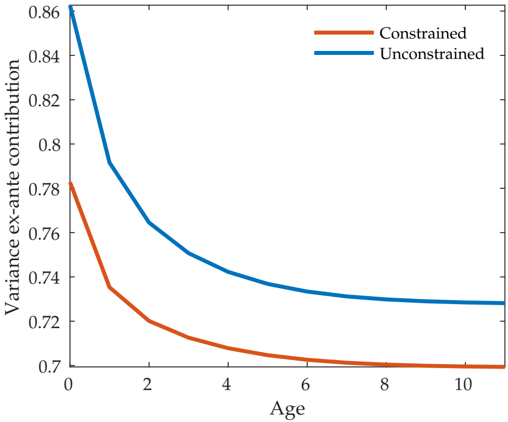


Figure: Unconstrained firms

Autocovariance constrained vs unconstrained - model



Ex-ante component variance contribution



Ex-ante expression

Firm potential estimation

- Is it firm potential that explains differences among constrained and unconstrained firms?
- Tentative way of estimating firms' potential

$$\mathbf{1}_{Constrained_t} = \sum_F \beta_F F_{it} + \sum_B \beta_B B_{it} + \sigma_i + \alpha_j + \epsilon_{it}$$

- Check R^2 with and without firm fixed effects
  - With: 0.592
  - Without: 0.089
- Regress firm fixed effects on observables (eg. birth year, location)
- Non-explained part is potential?

Theoretical considerations

- We find evidence for both a non-financial and a financial factors side of cyclical behaviour.
- Can a standard heterogeneous firm model with financial frictions explain this? No!
- Ex-ante conditions matter more for unconstrained firms, as defined by Pugsley et al. (2018)
- Incorporating this fact into a theoretical model should enable us to show that a size story and financial factors story can explain cyclicality jointly

Structural model - overview

We largely follow Khan and Thomas (2013) in the exposition of our model.

- Representative household, chooses labour and consumption households
- Large number of firms, each producing a homogeneous output subject to collateral constraints firms
- Firm-specific productivity schedule with ex-ante and ex-post components. productivity
- No aggregate risk

Conclusions

- Evidence of financial accelerator mechanism in the data
- Size and financial conditions both matter for firm cyclicality but are orthogonal to each other
- Typical proxies for constraints have weak correlations with the firm's financial conditions
- Ex-ante conditions matter to explain the financial state of the firm

Next steps

- What are good proxies for constrained firms for other datasets? MPK?
- Do the quantitative model
- Check aggregate effects
- Work with your feedback

Cyclicality of Turnover

	Sales growth			
	(1)	(2)	(3)	(4)
[90,99] × GDP growth	0.002	0.001	-0.001	0.002
	(0.001)	(0.001)	(0.002)	(0.001)
[99,99.5] × GDP growth	-0.010**	-0.010***	-0.020***	-0.010***
	(0.004)	(0.004)	(0.005)	(0.004)
[99.5,100] × GDP growth	-0.017***	-0.016***	-0.028***	-0.017***
	(0.004)	(0.004)	(0.006)	(0.004)
leverage × GDP growth		0.011***		
		(0.002)		
dividends × GDP growth			0.002***	
			(0.000)	
liquidity ratio × GDP growth				-0.006**
				(0.003)
Observations	803,742	803,678	446,405	803,678
R-squared	0.024	0.025	0.031	0.024
Industry FE	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

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Cyclicality of Turnover

	Sales growth			
	(1)	(2)	(3)	(4)
[90,99] × GDP growth	0.002	0.003*	0.002	0.002
	(0.001)	(0.001)	(0.001)	(0.001)
[99,99.5] × GDP growth	-0.010**	-0.009**	-0.010**	-0.010**
	(0.004)	(0.004)	(0.004)	(0.004)
[99.5,100] × GDP growth	-0.017***	-0.016***	-0.016***	-0.017***
	(0.004)	(0.004)	(0.004)	(0.004)
Constrained × GDP growth		0.002**		
		(0.001)		
Const + potential const × GDP growth			0.001	
			(0.001)	
Constrained continuous × GDP growth				-0.001***
				(0.000)
Observations	803,742	803,742	803,742	803,326
R-squared	0.024	0.024	0.024	0.024
Industry FE	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

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Standard deviation and autocorrelation by age

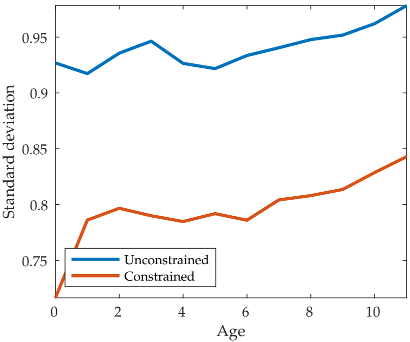

Figure: Standard deviation

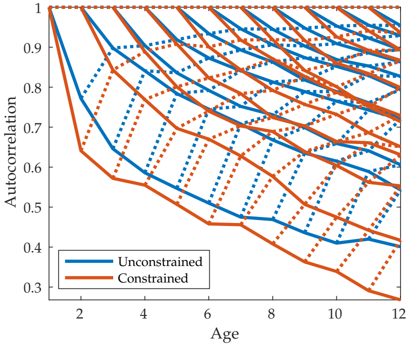

Figure: Autocorrelation

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Derivation of autocovariance formula (1/2)

Write stochastic processes in MA representation:

$$u_{i,t} = \rho_u^{t+1} u_{i,-1} + \sum_{k=0}^{a} \rho_u^k \theta_i$$

$$v_{i,a} = \rho_v^{a+1} v_{i,-1}$$

$$w_{i,a} = \sum_{k=0}^{a} \rho_w^k \varepsilon_{i,a-k} = \sum_{k=0}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} \quad 0 \leq j \leq a$$

So the level of log employment of firm i at age a is:

$$\ln n_{i,a} = \rho_u^{a+1} u_{i,-1} + \sum^{a} \rho_u^k \theta_i + \rho_v^{a+1} v_{i,-1} + \sum_{i=1}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{i=1}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} + z_{i,a}$$

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Derivation of autocovariance formula (2/2)

Then the autocovariance of log employment at age a and $a-j$ for $j \geq 0$ is:

$$\begin{aligned}
\mathrm{Cov}\left[\log n_{i,a}, \log n_{i,a-j}\right] &= \left(\sum_{k=0}^{a} \rho_u^k\right) \sigma_\theta^2 \left(\sum_{k=0}^{a-j} \rho_u^k\right) + \rho_u^{a+1} \sigma_{\tilde{u}}^2 \rho_u^{a-j+1} + \rho_v^{a+1} \sigma_{\tilde{v}}^2 \rho_v^{a-j+1} \\
&+ \mathrm{Cov}\left[\rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}, \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}\right] + \mathbf{1}_{\{j=0\}} \sigma_z^2 \\
&= \sigma_\theta^2 \left(\sum_{k=0}^{a} \rho_u^k\right) \left(\sum_{k=0}^{a-j} \rho_u^k\right) + \sigma_{\tilde{u}}^2 \rho_u^{2(a+1)-j} + \sigma_{\tilde{v}}^2 \rho_v^{2(a+1)-j} + \sigma_\varepsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \mathbf{1}_{\{j=0\}} \sigma_z^2
\end{aligned}$$

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Ex-ante contribution

$$\frac{\text{Ex-ante variance}}{\text{Total variance}}(\log n_{i,a}) = \frac{\left(\sum_{k=0}^{a} \rho_u^k\right)^2 \sigma_\theta^2 + \rho_u^{2(a+1)} \sigma_{\hat{u}}^2 + \rho_v^{2(a+1)} \sigma_{\hat{v}}^2}{\left(\sum_{k=0}^{a} \rho_u^k\right)^2 \sigma_\theta^2 + \rho_u^{2(a+1)} \sigma_{\hat{u}}^2 + \rho_v^{2(a+1)} \sigma_{\hat{v}}^2 + \sigma_\epsilon^2 \sum_{k=0}^{a} \rho_w^{2k} + \sigma_z^2}$$

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Households

A representative household solves the following recursive maximisation problem

$$V(k) = \max_{c,l,k'} \left\{ U(c,l) + \beta \mathbb{E} V(k') \right\}$$

subject to:

$$c + k' = (1+r)k + \omega l + D$$

Steady state FOCs that pin down wage and interest rates:

$$(1+r) = \frac{1}{\beta}$$

$$\omega = \frac{U_l(c,l)}{U_c(c,l)}$$

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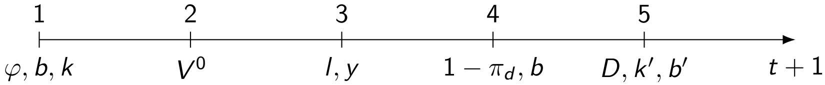
Firm side - main components

The main components of the firm side are:

1. Incumbents
   - Timing
   - Financial constraints
   - Productivity behaviour
   - Firm decisions
2. (Potential) entrants

Within-period timing

The within-period timing of an incumbent firm can be illustrated as follows:



1. Firms observe their idiosyncratic productivity φ, current stock of debt b and capital k
2. Choose whether to continue to production stage or exit the market according to $V^0 = \max(V^1, 0)$
3. Choose labour input and production
4. All firms that are hit by the stochastic, exogenous death shock d repay outstanding debt b and exit
5. Conditional on survival, firms choose their investment k' and borrowing b' subject to borrowing constraint $b' \leq \xi k$

Production stage

- Consider firms that choose to stay
- Their profits are given by $\pi = \varphi k^{\alpha} l^{\eta} - \omega l$
- Therefore, the optimal labour choice is

$$l(k, \varphi) = \left(\frac{\upsilon\varphi}{\omega} k^{\alpha}\right)^{\frac{1}{1-\upsilon}}$$

- And hence profits are:

$$\pi(k, \varphi) = \frac{k^{\frac{\alpha}{1-\upsilon}} \varphi^{\frac{1}{1-\upsilon}}}{\omega^{\frac{\upsilon}{1-\upsilon}}} \left(\upsilon^{\frac{\upsilon}{1-\upsilon}} - \upsilon^{\frac{1}{1-\upsilon}}\right)$$

Death shock and value of the firm

- After the production stage, firms might exit exogenously with probability π_d
- In that case its value is equal to its current cash-on-hand x
- Can write expected value of the firm at this stage as:

$$V^1(x, \varphi) = \pi_d x + (1 - \pi_d) V^2(x, \varphi)$$

where x is current cash-on-hand of the firm. Defined as:

$$x \equiv \pi(k, \varphi) + (1 - \delta)k - b - c_f$$

Survivors

Surviving firms face the following optimisation problem:

$$V^2(x, \varphi) = \max_{k', b', D} \left[D + E_{\varphi'|\varphi} \Lambda V^0 \left(x', \varphi' \right) \right]$$

$$\text{s.t.}$$

$$D \equiv x + qb' - k' \geq 0$$

$$b' \leq \theta k'$$

$$x' = \pi(k', \varphi') + (1 - \delta)k' - b' - c_f$$

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Model - Firm specific productivity

Use the same stochastic process for individual firm productivity as in reduced form model:

$$\ln \varphi_{i,t} = u_{i,t} + v_{i,t} + w_{i,t} + z_{i,t} \tag{1}$$

$$\begin{array}{llllr}
u_{i,t} = & \rho_u u_{i,t-1} + \theta_i, & u_{i,-1} \sim iid\left(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}\right), & \theta_i \sim iid\left(\mu_\theta, \sigma^2_\theta\right), & |\rho_u| \le 1 \\
v_{i,t} = & \rho_v v_{i,t-1}, & v_{i,-1} \sim iid\left(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}\right), & & |\rho_v| \le 1 \\
w_{i,t} = & \rho_w w_{i,t-1} + \varepsilon_{i,t}, & w_{i,-1} = 0, & \varepsilon_{i,t} \sim iid\left(0, \sigma^2_\varepsilon\right), & |\rho_w| \le 1 \\
z_{i,t} \sim & iid\left(0, \sigma^2_z\right) & & &
\end{array}$$

Note the large number of exogenous state variables: $[u_{i,t}, v_{i,t}, w_{i,t}, z_{i,t}]$. But we can reduce this somewhat:

- Assume that $\rho_v = \rho_w$. Then we only need to keep track of $w_{i,t} + v_{i,t}$. Call that combined variable $o_{i,t}$
- Note that $z_{i,t}$ is purely transitory and thus past values do not affect the decision of the firm.

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What do we need to keep track of then?

With the current setup a firm bases its decisions on:

1. Endogenous state: Cash on hand, call it $x_{i,t}$, i.e. profits plus non-depreciated capital minus the debt the firm has to pay back minus the fixed cost of production.
2. Exogenous states: Productivity components $[u_{i,t}, o_{i,t}]$

Thus we need a grid of:

1. $x_{i,t}$. Endogenous state variable, as usual.
2. u_i. Initial condition for ex-ante component away from zero
3. θ_i. Initial condition for permanent ex-ante component.
4. $o_{i,t}$. Persistent ex-post shocks.

Note: We need both a grid for u and θ as we are otherwise not able to predict tomorrow's value of u which matters for decisions.

Model - Entry and exit

- Fixed measure of potential entrants, M_e
- Uniformly distributed over b and k
- Stochastic productivity component distributed according to (1)
- Entrants need to pay entry cost f_e
- Entry takes place at the end of the period, start operating in the next period, given (x_0, φ_0)

Model - Financial sector

- Perfectly competitive financial intermediary providing loans
- Default costs: Lender can only recover fraction χ of the remaining value or b' if smaller.
- Firm-specific interest rates are being determined according to zero expected profits condition:

$$q(k', b', \varphi')b' = \beta \mathbb{E}_{\varphi'|\varphi}\Big[\mathbf{1}_{def}(x', \varphi')\min\big(b', \chi(1-\delta)k'\big) \\ + \big(1 - \mathbf{1}_{def}(x', \varphi')\big)b'\Big]$$

where $\mathbf{1}_{def}(x', \varphi')$ is an indicator variable equal to one if the firm defaults in state (x', φ').